



ELSEVIER

# Elimi

## Abstract

We sugg  
rance. Th  
ed. Such  
with this st  
© 2004 Else

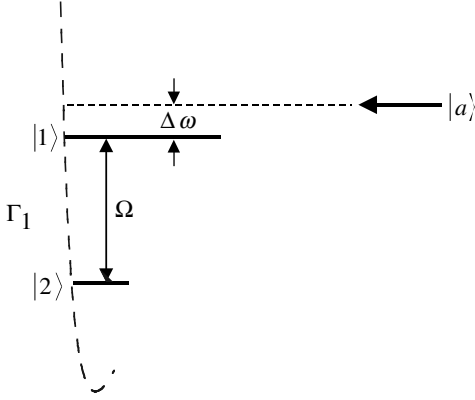
CS: 3-10

in rice  
dicates  
quasi-b  
close to  
concent  
for this  
losses as  
bond in  
et al [3]  
body res

In the  
[4] the  
lecular  
to em

\*Pres  
Univer  
Tel.: +6  
E-mail: [dyavuz@stanfordalumni.org](mailto:dyavuz@stanfordalumni.org) (D.D. Yavuz)

symmetric molecules this transition is



Rabi frequency. Without the coupling field, the atoms form quasi-bound molecules and, due to the large decay rate of these molecules, the atomic condensate experiences loss. With the coupling field, there is destructive interference in two quantum paths that form the quasi-bound molecules. In analogy with EIT for light fields, perfect interference is obtained only for the ideal case of  $\Gamma_2 = 0$ . The atomic condensate experiences no loss on the exact Feshbach resonance and the bandwidth of the transparency is determined by the intensity of the coupling field.

Pertinent prior work includes general formalisms that describe laser-assisted, electron-atom scattering [6] and cold-atom collisions in the presence of one or more resonant photoassociation lasers [7,8]. Van Abeleen and Verhaar [9] have suggested a possible loss mechanism when a Feshbach resonance is crossed rapidly with high ramp speed of the magnetic field. The schematic of Fig. 1 has been suggested by several authors as an efficient way to convert an atomic condensate to a molecular condensate [10,11]. Drummond et al. [12] has shown how stimulated Raman adiabatic passage (STIRAP) can be used to form a molecular condensate by Raman-coupling the free atomic state to a bound molecular state. Dynamics

between coupled atomic and molecular condensates have been discussed by Heinzen et al. [13] and Holland and Kokkelmans [14]. Coherent oscillations between atomic and molecular condensates have been observed by the Wieman group [15]. The formation of a molecular BEC has been recently demonstrated [16,17].

We proceed by using the mean field approach [18–20]. Following Fig. 1, the coupled, Gross-Pitaevskii equations for the atomic condensate field,  $\psi_a$ , the molecular condensate field for the quasi-bound state,  $\psi_1$ , and the molecular condensate field for the bound state,  $\psi_2$ , are:

$$i\hbar \frac{\partial \psi_a}{\partial t} = V_a \psi_a + \left( \sum_i g_{ai} |\psi_i|^2 \right) \psi_a + 2\alpha^* \hbar \psi_1 \psi_a^*, \quad (1a)$$

$$i\hbar \frac{\partial \psi_1}{\partial t} + i\hbar \frac{\Gamma_1}{2} \psi_1 = V_1 \psi_1 + \left( \sum_i g_{1i} |\psi_i|^2 \right) \psi_1 + \alpha \hbar \psi_a^2 + \hbar \Omega \psi_2, \quad (1b)$$

$$i\hbar \frac{\partial \psi_2}{\partial t} + i\hbar \frac{\Gamma_2}{2} \psi_2 = V_2 \psi_2 + \left( \sum_i g_{2i} |\psi_i|^2 \right) \psi_2 + \hbar \Omega \psi_1. \quad (1c)$$

Here, to simplify what follows, we assume all mean-field amplitudes to be spatially uniform. The quantities  $\Gamma_1$  and  $\Gamma_2$  are the decay rates,  $V_i$  are the uniform potentials (including the magnetic field contributions and resonance detunings for zero magnetic field) seen by each specie,  $\Omega$  is the Rabi frequency of the coupling field that couples molecular states, and the coupling coefficient  $\alpha$  describes the process that converts atoms to quasi-bound molecules (Feshbach process).  $g_{ij}$  are the self- and cross-interaction constants between the  $i$ th and  $j$ th specie and are related to background scattering length through the relation  $g_{ij} = 4\pi\hbar^2 a_{ij}/m$ . These equations have been used by many authors to describe coherent atomic and molecular condensate dynamics [3,10,11]. Working in interaction picture, we expand mean-field quantities as:  $\psi_a(t) = \phi_a(t)e^{-i\omega_a t}$ ;  $\psi_1(t) = \phi_1(t)e^{-i\Delta\omega t} e^{-i\omega_1 t}$ ; and  $\psi_2(t) = \phi_2(t)e^{-i\Delta\omega t} e^{-i\omega_2 t}$ . The energies of the states are defined as:  $\hbar\omega_a = V_a + \sum_i g_{ai} |\phi_i(0)|^2$ ,

$\hbar\omega_1 = V_1 + \sum_i g_{1i} |\phi_i(0)|^2$ , and  $\hbar\omega_2 = V_2 + \sum_i g_{2i} |\phi_i(0)|^2$ ;  $\Delta\omega = (2\omega_a - \omega_1)$  is the detuning from the Feshbach resonance. With these definitions, the coupled equations are:

$$i \frac{\partial \phi_a}{\partial t} = 2\alpha^* \phi_1 \phi_a^* + \frac{1}{\hbar} \left[ \sum_i g_{ai} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_a, \quad (2a)$$

$$\begin{aligned} \left( \Delta\omega + i \frac{\Gamma_1}{2} \right) \phi_1 + i \frac{\partial \phi_1}{\partial t} \\ = \alpha \phi_a^2 + \frac{\Omega}{2} \phi_2 + \frac{1}{\hbar} \left[ \sum_i g_{1i} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_1, \end{aligned} \quad (2b)$$

$$\begin{aligned} \left( \Delta\omega + i \frac{\Gamma_2}{2} \right) \phi_2 + i \frac{\partial \phi_2}{\partial t} \\ = \frac{\Omega^*}{2} \phi_1 + \frac{1}{\hbar} \left[ \sum_i g_{2i} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_2. \end{aligned} \quad (2c)$$

In the above equations, we have taken the coupling field to be on exact resonance with the molecular states and used the rotating-wave approximation. For simplicity, we have taken  $\Gamma_1$  to be a constant, and neglected the density dependent decay of the atomic and molecular condensates due to atom–molecule and molecule–molecule collisions. The inclusion of these decay terms do not change our results significantly. From Eqs. (2), with  $\Gamma_1 = \Gamma_2 = 0$ , we obtain the conservation condition for the total atom number:  $\frac{\partial}{\partial t} (|\phi_a|^2 + 2|\phi_1|^2 + 2|\phi_2|^2) = 0$ .

We proceed by numerically solving Eqs. (2) with the parameters of an MIT experiment [2,3]. We consider the 907 G Feshbach resonance in a Na atomic condensate with  $|\phi_a(0)|^2 = 5 \times 10^{14} / \text{cm}^3$ ,  $a_{aa} = 3.4$  nm, and  $\alpha = 4.11 \times 10^{-5}$  (mks). We take  $\Gamma_2 = 0.01$  Hz [11], consider an on-resonance excitation ( $\Delta\omega = 0$ ), and take  $\Gamma_1/2\pi = 0.3$  MHz. The choice of  $\Gamma_1$  is consistent within an order of magnitude to the estimated rates of van Abeleen et al. [9], Yurovsky et al. [3] and Timmermans et al. [19]. We also assume  $\phi_1(0) = \phi_2(0) = 0$ . Although the atom–molecule and molecule–molecule interaction constants are not known, our results are

insensitive to their exact values. In Fig. 2, the normalized density of the atomic condensate,  $|\phi_a|^2$ , is plotted vs. time for  $\Omega = 0$  and  $\Omega/2\pi = 1$  MHz, respectively. Without the coupling field, the condensate decays very rapidly; the loss is due to the formation of the quasi-bound molecules within the condensate, followed by the inelastic decay of these molecules. With the coupling field on, due to destructive interference, the loss to the atomic condensate is eliminated. The initial decrease in the density is due to the production of bound molecules, which is essential for creating interference. This phenomenon is in the spirit of the preparation energy for EIT with light fields [21].

The last terms on the right-hand side of Eqs. (2) are a density-dependent frequency shift of the mean-field amplitudes. We now proceed with the analytical interpretations of the numerical results of Fig. 2 and neglect the frequency shifts in Eqs. (2). We assume  $\frac{\partial \phi_1}{\partial t} \ll \Gamma_1 \phi_1$  and  $\frac{\partial \phi_2}{\partial t} \ll \frac{|\Omega|^2}{2\Gamma_1} \phi_2$  in Eqs. (2b) and (2c) to obtain expressions for the molecular field amplitudes:

$$\phi_1 = \frac{2\alpha(2\Delta\omega + i\Gamma_2)}{(2\Delta\omega + i\Gamma_1)(2\Delta\omega + i\Gamma_2) - |\Omega|^2} \phi_a^2, \quad (3a)$$

$$\phi_2 = \frac{2\alpha\Omega^*}{(2\Delta\omega + i\Gamma_1)(2\Delta\omega + i\Gamma_2) - |\Omega|^2} \phi_a^2. \quad (3b)$$

With these expressions, the differential equation for the atomic mean field amplitude (Eq. (2a)) is:

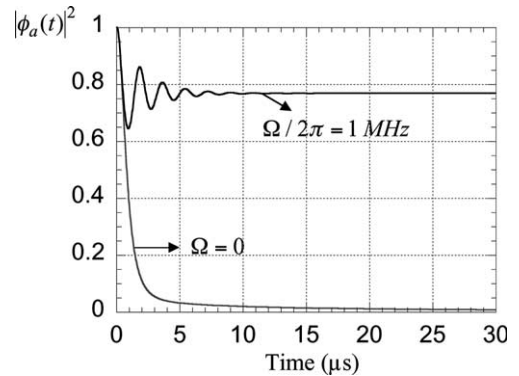


Fig. 2. The normalized density of the atomic condensate,  $|\phi_a|^2$ , vs. time for  $\Omega = 0$  and  $\Omega/2\pi = 1$  MHz. With the coupling field on, the decay of the atomic condensate is eliminated.

$$i \frac{\partial \phi_a}{\partial t} = \frac{4|\alpha|^2(2\Delta\omega + i\Gamma_2)}{(2\Delta\omega + i\Gamma_1)(2\Delta\omega + i\Gamma_2) - |\Omega|^2} |\phi_a|^2 \phi_a$$

$$\equiv \frac{4\pi\hbar}{m} a(\Delta\omega) |\phi_a|^2 \phi_a, \quad (4)$$

where  $a(\Delta\omega)$  is the complex scattering length for the atomic condensate (without the background contribution). The imaginary part of  $a(\Delta\omega)$  determines the loss to the condensate and is plotted vs. frequency in Fig. 3(a) for the parameters of the numerical simulation of Fig. 2. With the coupling field, the loss profile is double-peaked and is zero on the exact Feshbach resonance (previous maximum). By choosing the intensity of the coupling field, the interference profile may be either narrower or wider than the decay width,  $\Gamma_1$ . The real part of  $a(\Delta\omega)$  is the change in the atom–atom interaction energy due to the Feshbach interaction and is plotted versus frequency in Fig. 3(b) [22].

As seen from Fig. 3(a), perfect transparency is only obtained on exact Feshbach resonance. Due to the nonlinear frequency shifts of Eqs. (2), an

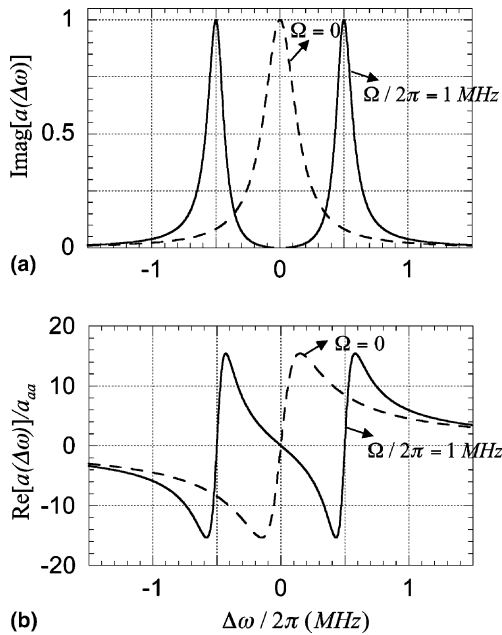


Fig. 3. (a) The normalized imaginary part of the scattering length. With the coupling field on, there is no loss on exact Feshbach resonance. (b) The real part of the scattering length normalized to its background value.

atomic condensate which is initially tuned to the exact Feshbach resonance will go in and out of the resonance during coherent interactions and the transparency will be degraded. One way to eliminate these shifts is to bound the minimum intensity of the coupling field thereby limiting the number of generated molecules. From Eqs. (3), this minimum intensity bound is:

$$|\Omega|^2 \gg 4|\alpha|^2 |\phi_a|^2. \quad (5)$$

This restriction might be overcome by tracking the resonance using a time varying magnetic field or a frequency-chirped coupling laser.

As noted above, throughout this paper, for simplicity, we have taken  $\Gamma_1$  to be constant and neglected density dependent decay of the atomic and molecular condensates due to atom–molecule collisions. With the inclusion of these decay terms, Eqs. (2) read:

$$i \frac{\partial \phi_a}{\partial t} + i\gamma |\phi_1|^2 \phi_a$$

$$= 2\alpha^* \phi_1 \phi_a^* + \frac{1}{\hbar} \left[ \sum_i g_{ai} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_a,$$

$$\left( \Delta\omega + i \frac{\Gamma_1}{2} + i\gamma |\phi_a|^2 \right) \phi_1 + i \frac{\partial \phi_1}{\partial t}$$

$$= \alpha \phi_a^2 + \frac{\Omega}{2} \phi_2 + \frac{1}{\hbar} \left[ \sum_i g_{1i} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_1,$$

$$\left( \Delta\omega + i \frac{\Gamma_2}{2} \right) \phi_2 + i \frac{\partial \phi_2}{\partial t}$$

$$= \frac{\Omega^*}{2} \phi_1 + \frac{1}{\hbar} \left[ \sum_i g_{2i} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_2, \quad (6)$$

where  $\gamma$  is the atom–molecule state changing collision rate [3,19]. In Eq. (6),  $\Gamma_1$  represents any other unknown loss mechanism of the fragile quasi-bound state. Numerically solving Eq. (6), with  $\gamma \approx 10^{-9}–10^{-10} \text{ cm}^3/\text{s}$  [3,19], and with all the other parameters identical, we find that the results of Fig. 2 practically remain unchanged.

In our formalism, for simplicity, we have not included an upper electronic molecular state, which is necessary for two-photon coupling the quasi-bound and bound molecular states in symmetric molecules. For the numerical simulations of

Fig. 2, the inclusion of such a state doesn't change the results significantly as long as the detuning from this state is larger than 10 times the decay width of this state. Strong Raman coupling for these conditions require laser power densities of about 10 kW/cm<sup>2</sup>. Tuning closer to the upper electronic state reduces the laser intensity requirement, however degrades transparency. The amount of degradation can be found by adiabatically eliminating the mean-field amplitude of the upper electronic state and substituting the relevant quantities in Eq. (4) with their effective values. These substitutions are:  $\Omega \rightarrow \Omega_1\Omega_2^*/(2\delta\omega + i\Gamma_3)$ ,  $\Gamma_1 \rightarrow \Gamma_1 - \text{Imag}[\Omega_1^2/(2\delta\omega + i\Gamma_3)]$ , and  $\Gamma_2 \rightarrow \Gamma_2 - \text{Imag}[\Omega_2^2/(2\delta\omega + i\Gamma_3)]$ . Here  $\Omega_1$  and  $\Omega_2$  are the Rabi frequencies of the two driving lasers,  $\delta\omega$  is the detuning from upper state, and  $\Gamma_3$  is the decay width of this state.

Several recent papers discuss the importance of the noncondensate modes in the coupled condensate dynamics [14,23–25]. We believe that the inclusion of these modes will not change the results of this letter significantly since the quasi-bound state does not accumulate significant population.

In summary, we have suggested a technique to eliminate losses to an atomic condensate near a Feshbach resonance. The predictions of this letter can be realized within current experimental conditions. By coupling the quasi-bound state, the double-peaked interference profile of Fig. 3(a) should be readily observable. The atomic condensate experiences dispersion of the scattering length at the point of zero loss. Such a unique dispersive feature may find possible applications in nonlinear matter–wave interactions.

## Acknowledgements

I would like to thank Steve Harris for suggesting the problem and for many fruitful discussions and Zachary Dutton for an early suggestion. This work was supported by the OSD Multidisciplinary University Research Initiative Program (MURI), the US Army Research Office, the US Office of Naval Research, and the US Air Force Office of Scientific Research.

## References

- [1] S. Inouye, M.R. Andrews, J. Stenger, H.-J. Miesner, D.M. Stamper-Kurn, W. Ketterle, *Nature (London)* 392 (1998) 151.
- [2] J. Stenger, S. Inouye, M.R. Andrews, H.-J. Miesner, D.M. Stamper-Kurn, W. Ketterle, *Phys. Rev. Lett.* 82 (1999) 2422.
- [3] V.A. Yurovsky, A. Ben-Reuven, P.S. Julienne, C.J. Williams, *Phys. Rev. A* 60 (1999) R765.
- [4] S.E. Harris, *Phys. Rev. A* 66 (2002) 010701.
- [5] K.-J. Boller, A. Imamoglu, S.E. Harris, *Phys. Rev. Lett.* 66 (1991) 2593; M.O. Scully, M.S. Zubairy, *Quantum Optics*, Cambridge University Press, Cambridge, England, 1997.
- [6] N.J. Kylstra, C.J. Joachain, *Europhys. Lett.* 36 (1996) 657; N.J. Kylstra, C.J. Joachain, *Phys. Rev. A* 57 (1998) 412.
- [7] P.O. Fedichev, Y. Kagan, G.V. Shlyapnikov, T.M. Walraven, *Phys. Rev. Lett.* 77 (1996) 2913.
- [8] J.L. Bohn, P.S. Julienne, *Phys. Rev. A* 56 (1997) 1486; J.L. Bohn, P.S. Julienne, *Phys. Rev. A* 60 (1999) 414.
- [9] F.A. van Abeelen, B.J. Verhaar, *Phys. Rev. Lett.* 83 (1999) 1550.
- [10] M. Mackie, R. Kowalski, J. Javanainen, *Phys. Rev. Lett.* 84 (2000) 3803; M. Mackie, *Phys. Rev. A* 66 (2002) 043613.
- [11] S.J.J.M.F. Kokkelmans, H.M.J. Vissers, B.J. Verhaar, *Phys. Rev. A* 63 (2001) 031601.
- [12] P.D. Drummond, K.V. Kheruntsyan, D.J. Heinzen, R.H. Wynar, *Phys. Rev. A* 65 (2002) 063619.
- [13] D.J. Heinzen, R. Wynar, P.D. Drummond, K.V. Kheruntsyan, *Phys. Rev. Lett.* 84 (2000) 5029.
- [14] S.J.J.M.F. Kokkelmans, M.J. Holland, *Phys. Rev. Lett.* 89 (2002) 180401.
- [15] E.A. Donley, N.R. Claussen, S.T. Thompson, C.E. Wieman, *Nature (London)* 417 (2002) 529.
- [16] S. Jochim, M. Bartenstein, A. Altmeyer, G. Hendl, S. Riedl, C. Chin, J.H. Denschlag, R. Grimm, *Science* 302 (2003) 2101.
- [17] M. Greiner, C.A. Regal, D.S. Jin, *Nature* 426 (2003) 537.
- [18] F. Dalfovo, S. Giorgini, L.P. Pitaevskii, S. Stringari, *Rev. Mod. Phys.* 71 (1999) 463.
- [19] E. Timmermans, P. Tommasini, M. Hussein, A. Kerman, *Phys. Rep.* 315 (1999) 199.
- [20] E. Timmermans, P. Tommasini, R. Cote, M. Hussein, A. Kerman, e-print cond-mat/9805323.
- [21] S.E. Harris, Z.F. Luo, *Phys. Rev. A* 52 (1995) 929.
- [22] The expression for the scattering length is identical to that obtained in Ref. [4].
- [23] M. Mackie, K. Suominen, J. Javanainen, *Phys. Rev. Lett.* 89 (2002) 180403.
- [24] V.A. Yurovsky, A. Ben-Reuven, *Phys. Rev. A* 67 (2003) 043611.
- [25] T. Kohler, T. Gasenzer, K. Burnett, *Phys. Rev. A* 67 (2003) 013601.