

Electromagnetically induced transparency with broadband laser pulses

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We suggest a scheme to slow and stop broadband laser pulses inside an atomic medium using electromagnetically induced transparency. Extending the suggestion of Harris *et al.* [Phys. Rev. Lett. **70**, 552 (1993)], the key idea is to use matched Fourier components for the probe and coupling laser beams.

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Over the last decade, counterintuitive optical effects using electromagnetically induced transparency (EIT) have gained considerable attention [1–3]. The essence of EIT is to create a narrow transparency window in an otherwise opaque medium using quantum interference. Noting Fig. 1, in a three-state atomic medium, the quantum interference is achieved with a laser beam that couples states $|2\rangle$ and $|3\rangle$. An important feature of EIT is the steep dispersion experienced by the probe laser beam at the point of vanishing absorption. This steep dispersion is the essence of slow light [4–6], stopped light [7–9], and giant nonlinearities effective at low light levels [10–14].

An important practical application of EIT is to optical information processing. EIT provides a unique way to controllably delay and coherently store light pulses. However, the narrow transparency window of EIT puts stringent limitations on the bandwidth of the light pulses that can be slowed and stopped inside the medium. A key figure of merit that is usually discussed in this context is the time-delay-bandwidth product which is obtained by multiplying the bandwidth of the optical pulse with the delay time of the pulse while propagating through the EIT medium. The largest time-delay-bandwidth product that has been experimentally demonstrated using EIT is ≈ 5 . Recently, Dutton and colleagues [15] and Zubairy and colleagues [16] have suggested schemes to overcome this limitation. The key idea of these schemes is to spectrally disperse an input optical pulse and independently delay each Fourier component of the pulse. Experimental progress towards demonstrating these schemes has also been reported [17,18].

In this Rapid Communication we extend the suggestion of Harris *et al.* [19] and demonstrate a scheme that allows large time delays for large bandwidth optical pulses. The key idea of our scheme is to use matched Fourier components for the probe and coupling laser beams in an EIT medium. Before we proceed with a detailed description of our scheme, we would like to summarize the well-known results for the interaction of two laser beams with a three-state Λ system. Noting Fig. 1, in the perturbative limit where the probe laser beam Ω_p is much weaker than the coupling beam Ω_c , the susceptibility for the probe wave is

$$\chi(\omega_p) = 4 \frac{N|\mu_{13}|^2}{\epsilon_0 \hbar} \frac{\delta\omega}{2[2(\Delta + \delta\omega) + j\Gamma]\delta\omega - |\Omega_c|^2}. \quad (1)$$

Here μ_{13} is the dipole matrix element, Γ is the decay rate of the excited state, Ω_c is the Rabi frequency of the coupling

laser beam, and the detunings are defined as $\Delta = (\omega_3 - \omega_2) - \omega_c$, $\delta\omega = (\omega_2 - \omega_1) - (\omega_p - \omega_c)$. The susceptibility of Eq. (1) assumes infinite dephasing time of the $|1\rangle$ to $|2\rangle$ Raman transition. In Fig. 1, we plot the imaginary part of the susceptibility of Eq. (1) for $\Omega_c = \Gamma/3$ and $\Delta = -\Gamma$, $\Delta = 0$, and $\Delta = \Gamma$, respectively. In all cases, perfect transparency is achieved for exact two-photon resonance, $\delta\omega = 0$. Furthermore, even though the line shape becomes asymmetric for $\Delta \neq 0$, the steep dispersion is maintained at the point of vanishing absorption. The group velocity at $\delta\omega = 0$ is determined by the intensity of the coupling laser beam, and is independent of Δ .

Motivated by the results of Fig. 1, we consider a broad set of frequencies for the probe laser beam where each frequency has a matching component in the coupling laser beam such that the two-photon resonance condition is maintained. Noting Fig. 2, EIT and therefore slow light is achieved in parallel channels for each of the frequencies of the probe laser beam. Below we will show that (i) similar to the traditional EIT scheme of Eq. (1), the group velocity is controlled by the intensity of the coupling laser beam. Here, however, the important quantity is the total intensity, which is obtained by summing the intensities of the coupling laser in all channels. (ii) In the ideal limit of infinite dephasing time for the Raman transition, the group velocity and therefore the delay time for the probe pulse is independent of its

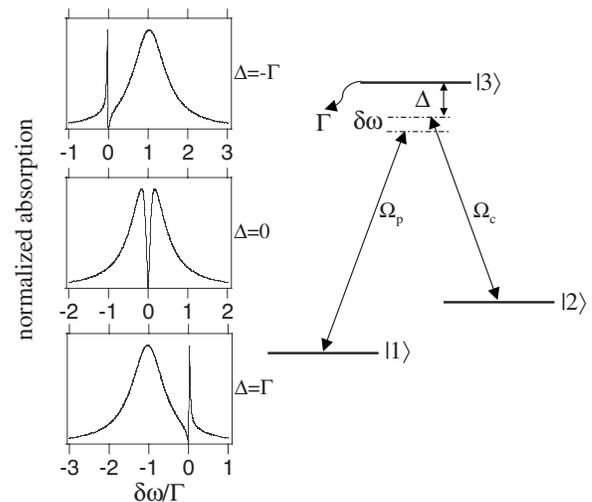


FIG. 1. The susceptibility for the probe laser beam as a function of two-photon detuning $\delta\omega$ for $\Delta = -\Gamma$, $\Delta = 0$, and $\Delta = \Gamma$, respectively. For all cases, perfect transparency is achieved when the two-photon resonance condition is satisfied, $\delta\omega = 0$.

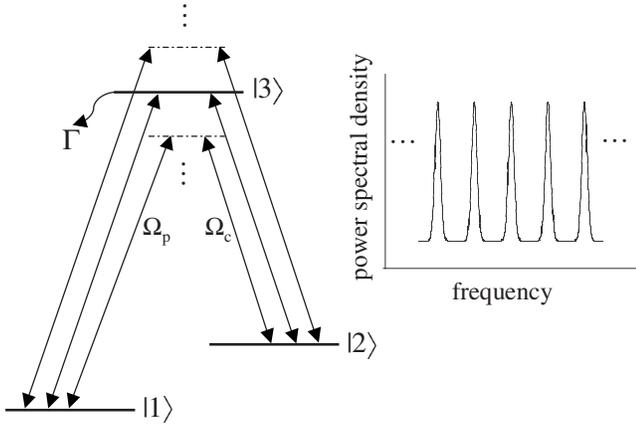


FIG. 2. The suggestion of our scheme. We consider the propagation of a broad set of frequencies for the probe laser beam where each frequency has a matching component in the coupling laser beam such that the two-photon resonance condition is maintained. EIT is achieved in parallel channels for each of the frequency components.

bandwidth. As a result it becomes possible to obtain a large time-delay-bandwidth product.

We proceed with a detailed description of our scheme. We expand the total electric field as $E(z,t) = \text{Re}[E_p(z,t) \times \exp(j\omega_p t - jk_p z)] + \text{Re}[E_c(z,t) \exp(j\omega_c t - jk_c z)]$ where $\omega_p = \omega_3 - \omega_1$ and $\omega_c = \omega_3 - \omega_2$. The detunings from the respective transitions are therefore incorporated into the slowly varying envelopes E_p and E_c . We define the Rabi frequencies for the respective transitions $\Omega_p = E_p \mu_{13} / \hbar$, $\Omega_c = E_c \mu_{23} / \hbar$. Working in local time, $\tau = t - z/c$, the Schrodinger equation for the probability amplitudes of the three states within the rotating wave approximation is

$$\begin{aligned} \frac{\partial c_1}{\partial \tau} &= \frac{j}{2} \Omega_p c_3, \\ \frac{\partial c_2}{\partial \tau} &= \frac{j}{2} \Omega_c c_3, \\ \frac{\partial c_3}{\partial \tau} + \frac{\Gamma}{2} c_3 &= \frac{j}{2} \Omega_p^* c_1 + \frac{j}{2} \Omega_c^* c_2. \end{aligned} \quad (2)$$

The decay processes are assumed to be to states outside the system. With the probability amplitudes calculated with Eq. (2), the slowly varying envelope Maxwell's equations for the two laser beams are

$$\begin{aligned} \frac{\partial \Omega_p}{\partial z} &= -\frac{j}{\hbar} \eta \omega_p N |\mu_{13}|^2 c_1 c_3^*, \\ \frac{\partial \Omega_c}{\partial z} &= -\frac{j}{\hbar} \eta \omega_c N |\mu_{23}|^2 c_2 c_3^*, \end{aligned} \quad (3)$$

where N is the atomic density and $\eta = \sqrt{\mu/\epsilon_0}$. Throughout the rest of this paper, we analyze the propagation of a broad set of frequencies for the probe and coupling laser beams through an atomic system defined by the above coupled

equations. We solve Eqs. (2) and (3), with the initial condition that all the atoms start in the ground state $|1\rangle$, and the boundary condition at the beginning of the cell ($z=0$) for the two laser beams: $\Omega_p(z=0, \tau) = \Omega_{p0}(\tau) f(\tau)$, $\Omega_c(z=0, \tau) = \Omega_{c0}(\tau) f(\tau)$. Here $\Omega_{p0}(\tau)$ and $\Omega_{c0}(\tau)$ are long envelopes that allow adiabatic preparation of the medium, and $f(\tau)$ defines the broad set of frequencies that are considered, $f(\tau) = \sum_q f_q \exp(j\omega_q \tau)$. The function $f(\tau)$ is dimensionless and is normalized such that $\sum_q |f_q|^2 = 1$. As we will show below, the dynamics of the EIT medium will be determined by $\Omega_{p0}(\tau)$, and $\Omega_{c0}(\tau)$, and will largely be independent of the time variation that is common to both fields, $f(\tau)$.

Before proceeding with the analytical results, we present numerical simulations in a real atomic system with parameters similar to current experiments. We choose our atomic medium to be ^{87}Rb with $\Gamma = 2\pi \times 6.06$ MHz at an atomic density of $N = 10^{12}/\text{cm}^3$. The two Raman states are $|1\rangle \equiv |F=1, m_F=0\rangle$, and $|2\rangle \equiv |F=2, m_F=0\rangle$ hyperfine states of the ground electronic state $5S_{1/2}$. The excited state is chosen to be $|3\rangle \equiv |F'=2, m_{F'}=1\rangle$ of $5P_{3/2}$ ($D2$ line). We assume the two laser beams to have the same circular polarization. We take $f(\tau)$ to consist of 31 equally spaced frequencies, $\omega_q = q\omega_m$, with $\omega_m = \Gamma/4$, and $q = -15, -14, \dots, 15$, and choose their amplitudes and phases such that they synthesize a square wave with a period of $0.66 \mu\text{s}$. The total spectral content of the probe pulse is therefore ≈ 45 MHz which is broad when compared with the linewidth of the excited state. As shown in Fig. 3, we assume a Gaussian envelope Ω_{p0} for the probe laser beam with a Gaussian width of $12 \mu\text{s}$. The envelope for the coupling laser beam, Ω_{c0} , smoothly turns on to its peak value of $\Omega_{c0, \text{peak}} = \Gamma/3$ and stays constant (not shown in Fig. 3). To assure adiabatic preparation of the medium, the coupling laser beam is turned on before the probe laser beam (counterintuitive pulse sequence). We numerically solve Eqs. (2) and (3) on a space-time grid using the method of lines.

In Fig. 3, we demonstrate slowing of broadband pulses with our scheme. We plot the envelope of the probe laser beam at $z=0$ and $z=5$ mm, respectively. The insets zoom in on the central portion of the waveform to display the synthesized square wave. The probe pulse propagates with a group velocity of $v_g = 58$ m/s and is delayed by $84 \mu\text{s}$ at the end of the medium. The shape of the square waveform is almost perfectly preserved demonstrating that all Fourier components of the input pulse propagate without significant loss and phase-shift. The time-delay-bandwidth product that is achieved in this simulation is $\approx 10^3$. The coupling laser beam (not shown in Fig. 3) propagates without significant change through the medium.

In Fig. 4, we demonstrate stopping of broadband pulses. Here, differing from the simulation of Fig. 3, we take the coupling laser beam envelope to smoothly turn off for a duration of $100 \mu\text{s}$ in Fig. 4(a) and $250 \mu\text{s}$ in Fig. 4(b), and then turn back on again. The probe beam is therefore stored in the medium for a controllable amount of time and then released. For both cases, the released pulse contains all the Fourier components of the input pulses with relative phases and amplitudes preserved. The inset in Fig. 4(b) is a zoom in on the central portion of the probe envelope that shows the

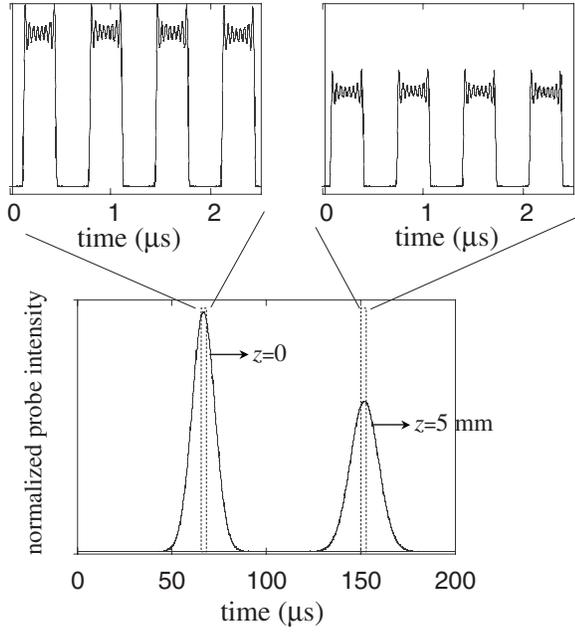


FIG. 3. Slowing of broadband light pulses inside an ^{87}Rb with a density of $N=10^{12}/\text{cm}^3$. At the beginning of the cell, the probe beam is assumed to be $\Omega_p(z=0, \tau) = \Omega_{p0}(\tau)f(\tau)$, where Ω_{p0} is a long envelope with a Gaussian width of $12 \mu\text{s}$, and $f(\tau)$ is a rapidly varying square time waveform. The bottom plot shows the envelopes of the probe beam at $z=0$ and $z=5 \text{ mm}$, respectively. The probe beam is delayed by $84 \mu\text{s}$. The insets zoom in on the central portion of the waveform to display the rapidly varying square wave. The time-delay-bandwidth product that is achieved in this simulation is $\approx 10^3$.

square-wave temporal structure demonstrating coherent storage of the broadband probe pulse.

We now proceed with a perturbative analytical solution of Eqs. (2) and (3) to get an insight into the results of Figs. 3 and 4. We follow closely the formalism of Eberly and colleagues [20]. We proceed perturbatively, and take the probe beam to be much weaker than the coupling beam. With counterintuitive pulse sequence, the medium can be prepared in the dark state when the following two-field adiabatic condition is satisfied:

$$\left| \Omega_p \frac{\partial \Omega_c}{\partial \tau} - \Omega_c \frac{\partial \Omega_p}{\partial \tau} \right| \ll |\Omega_c|^3. \quad (4)$$

With $\Omega_p(z, \tau) = \Omega_{p0}(z, \tau)f(\tau)$ and $\Omega_c(z, \tau) = \Omega_{c0}f(\tau)$ (Ω_{c0} independent of space and time), the two-field adiabatic condition of Eq. (4) reduces to

$$\frac{1}{|\Omega_{c0}|^2} \left| \frac{\partial \Omega_{p0}}{\partial \tau} \right| \ll |f(\tau)|. \quad (5)$$

We note that the adiabatic criteria of Eq. (5) is independent of the bandwidth of the time function $f(\tau)$. This is the key reason why the dynamics of the system is largely independent of $f(\tau)$, as long as Eq. (5) is satisfied. With the medium prepared adiabatically, the solution of the Schrodinger equation [Eq. (2)], including the first nonadiabatic correction to c_3 is

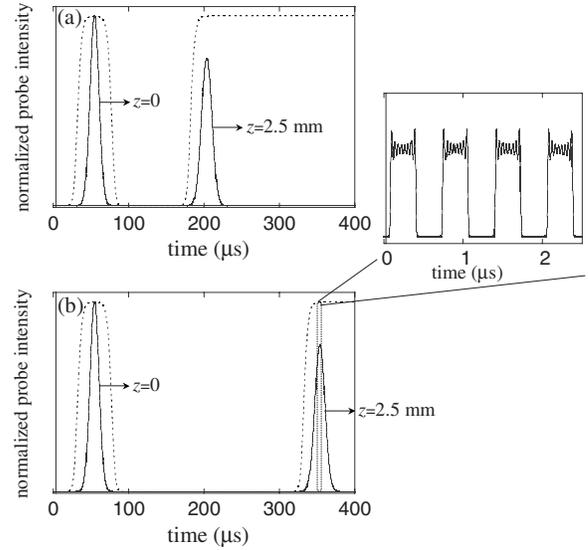


FIG. 4. Stopping of broadband light pulses using EIT. The probe pulse is stored for $100 \mu\text{s}$ in (a) and $250 \mu\text{s}$ in (b) and then released. Stopping of the probe pulse is achieved by smoothly turning down the intensity of the coupling laser beam. In both plots, the normalized intensity envelope of the probe pulse at $z=0$ and $z=2.5 \text{ mm}$ is plotted. The dotted line is the intensity envelope of the coupling laser beam. The inset in (b) is a zoom in on the central portion of the released pulse.

$$c_1 \approx 1, \quad c_2 \approx -\frac{\Omega_p^*}{\Omega_c}, \quad c_3 \approx j \frac{2}{\Omega_c} \frac{\partial}{\partial \tau} \left(\frac{\Omega_p^*}{\Omega_c} \right). \quad (6)$$

With the solution of Eq. (6), the propagation equation for the probe beam becomes

$$\begin{aligned} \frac{\partial \Omega_p}{\partial z} &= -\frac{2}{\hbar} \eta \omega_p N |\mu_{13}|^2 \frac{1}{\Omega_c^*} \frac{\partial}{\partial \tau} \left(\frac{\Omega_p}{\Omega_c} \right) \\ &\Rightarrow \frac{\partial \Omega_{p0}}{\partial z} |f(\tau)|^2 \\ &= -\frac{2}{\hbar} \eta \omega_p N |\mu_{13}|^2 \frac{1}{|\Omega_{c0}|^2} \frac{\partial \Omega_{p0}}{\partial \tau}. \end{aligned} \quad (7)$$

Following Ref. [20] we make the following change of variable in Eq. (7), $\xi(\tau) = \int_0^\tau |f(\tau')|^2 d\tau'$. With this transformation, the analytical solution of Eq. (7) can be found:

$$\Omega_{p0}(z, \tau) = \tilde{\Omega}_{p0}(\xi - z/\tilde{V}_g), \quad \frac{1}{\tilde{V}_g} = \frac{2\eta\omega_p N |\mu_{13}|^2}{\hbar |\Omega_{c0}|^2}, \quad (8)$$

where $\tilde{\Omega}_{p0}(\xi) = \Omega_{p0}(0, \tau)$. Using $f(\tau) = \sum_q f_q \exp(j\omega_q \tau)$ and $\sum_q |f_q|^2 = 1$, we have $\xi(\tau) = \tau + \epsilon(\tau)$ where $\epsilon(\tau) = \int_0^\tau \sum_q \sum_{q'} f_q f_{q'}^* \exp[j(\omega_q - \omega_{q'})\tau'] d\tau'$. For the broad set of frequencies considered as in Figs. 3 and 4, $\epsilon(\tau) \ll \tau$ and therefore $\tilde{\Omega}_{p0}(\xi) \approx \tilde{\Omega}_{p0}(\tau)$. Equation (8) shows that the probe pulse propagates without attenuation and with a group velocity of $\frac{1}{\tilde{V}_g} = \frac{1}{c} + \frac{1}{\tilde{V}_g}$ in agreement with the numerical results of Fig. 3. Remarkably, similar to the traditional EIT scheme of Eq. (1), the group velocity is determined by the intensity of

the coupling laser beam, $|\Omega_{c0}|^2$. As noted before, this groupvelocity (and therefore the time delay) is independent of the bandwidth of $f(\tau)$, assuming infinite dephasing time of the Raman transition.

Throughout this Rapid Communication we have assumed the ideal case of homogeneous broadening of the excited state. We expect our scheme to work in the presence of Doppler broadening, since our scheme is insensitive to the detuning from the excited state, as long as two-photon resonance condition is satisfied. As noted in the introduction, one possible application of our approach is to optical information processing. In contrast to other proposed schemes [15,16] our approach does not require spectral processing of the

probe pulse. However, a significant disadvantage of our approach is that it requires a time varying coupling laser with Fourier components exactly matched to the probe laser beam. Our approach may also find applications in achieving giant nonlinearities effective at the single-photon levels using EIT [10–14]. When compared with the narrow-bandwidth traditional EIT schemes, it may be advantageous to use larger bandwidth, and therefore higher peak power, single-photon pulses.

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