Contents lists available at ScienceDirect

**Optics Communications** 

journal homepage: www.elsevier.com/locate/optcom

# Far-off resonance conditional phase-shifter using the ac Stark shift

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#### ARTICLE INFO

Article history: Received 10 March 2009 Received in revised form 11 May 2009 Accepted 11 May 2009

PACS: 42.65.Pc 42.79.Ta 42.50.-p

#### ABSTRACT

We suggest a conditional phase-shifter that achieves a phase shift of  $\pi$  radians between two weak laser beams with a total energy density on the level of 1000 photons per atomic cross-section. The two laser beams interact through the simple nonlinear technique of ac Stark shifting the common ground state of a V-type system. We find that this switch can operate in the far-off resonance regime, with low absorption and high phase accumulation. Additionally, the bandwidth of this switch can be increased independently of the energy requirement.

Published by Elsevier B.V.

Interacting low-power laser beams is a subject of considerable attention in nonlinear and quantum optics [1-3]. Nonlinear interactions between weak beams can form optical switches with possible applications in all-optical information processing. Furthermore, if achieved at the single photon level, these interactions can also be used to entangle single photons, which may form the basis of a future photonic quantum computing device. In traditional nonlinear materials, the weakness of optical nonlinearities prohibit observing significant nonlinear effects between weak beams. Over the last decade, suggestions involving Electromagnetically Induced Transparency (EIT) have generated much enthusiasm in this field [4-11]. Recent experiments have demonstrated optical switching at  $\sim 10$  photons per atomic cross-section using EIT-based approaches [10,12]. Additionally, switching with optical instabilities has been demonstrated in an atomic vapor at less than one photon per atomic cross-section [13].

A well-known scheme for interacting laser beams is through the ac Stark shift of a common ground state [1–3]. Here, an intense laser beam can modify the refractive index experienced by a weak beam by changing its frequency detuning from a resonance. In this communication, we analyze this effect in an alkali atomic vapor where the two beams are far-off resonance from the excited electronic state. We find that a conditional phase shift on the order of radians can be obtained with an energy density around 1000 photons per atomic cross-section,  $\lambda^2/(2\pi)$ . Our results show that the scheme detailed here could operate as a simple, high-bandwidth, all optical switch with low absorption. Although the physics of what we are going to discuss is well-understood, to our knowledge,

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the possibility of constructing an ultra-low energy switch by using far-off resonance ac Stark shift has not been discussed before.

Before describing the scheme in detail, we note the several advantages of this approach when compared with similar optical switches utilizing other approaches: (1) This scheme does not require a strong coupling laser as is required by EIT. As a result, the total energy density requirement of our switch is at the 1000-photon level per atomic cross-section. (2) The bandwidth of our switch can be large and one can work with nanosecond time scale optical pulses. The bandwidth can be increased until the rotating-wave approximation breaks down at the expense of an increased density-length product. (3) For sufficiently large detuned beams, Doppler broadening becomes unimportant and as a result, our scheme is well suited for vapor cells. Due to these advantages, our approach may be particularly useful for constructing ultra-low energy, high-bandwidth all-optical switches with possible applications in current fiber-optic networks.

As shown in Fig. 1, we begin with a neutral alkali atomic medium containing a ground state  $|1\rangle$  and two excited states,  $|2\rangle$  and  $|3\rangle$ . A probe beam,  $E_p$ , and a switch beam,  $E_s$ , are tuned far-off resonance from the  $|1\rangle-|2\rangle$  and  $|1\rangle-|3\rangle$  transitions, respectively.<sup>1</sup> Without the switch beam, the weak probe laser will experience phase accumulation and absorption as determined by the linear susceptibility of the atomic medium. These quantities depend on the probe's frequency detuning from the atomic resonance,  $\Delta_p$ . When both the probe and switch propagate together through the medium, the detuning of the probe effectively changes. This is because the switch beam will ac Stark shift the common ground state

<sup>1</sup> In general, this phase-shifter scheme is not exclusive to V-systems. For example, one may tune both beams to the same lower and upper states in a two-level scheme. Then, the switch beam will ac Stark shift both the lower and upper states.



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**Fig. 1.** A switch beam,  $E_s$ , causes a nonlinear phase shift on a probe beam,  $E_p$ . The two beams travel collinearly through a V-type atomic medium. By itself, the probe accumulates phase based on the linear susceptibility of the atoms. When the switch beam is turned on, the common ground state  $|1\rangle$  experiences an ac Stark shift, changing the effective detuning. The plot indicates that ~1000 switch photons per atomic cross-section are required for a phase shift of  $\pi$  radians on the probe.

|1). In the presence of the switch beam, the susceptibility of the medium is modified to give:

$$\chi_p = \frac{N\mu_{12}^2}{\hbar\epsilon_0} \frac{1}{2(\varDelta_p + \delta_s) - i\Gamma},\tag{1}$$

where  $\delta_s = \frac{\Omega_c^2}{4\Delta_s}$  is the ac Stark shift of the ground state,  $\Omega_s$  is the Rabi frequency of the switch beam, N is the atomic density,  $\mu_{12}$  is the dipole matrix element between states  $|1\rangle$  and  $|2\rangle$ , and  $\Gamma$  is the transition linewidth. In the perturbative limit where  $\delta_s \ll \Delta_p$ , the nonlinear interaction between the switch and the probe can be described with a third-order  $\chi^{(3)}$  susceptibility by expanding Eq. (1). The polarization of the atomic medium at the probe laser frequency is then  $P_p = \epsilon_0 \chi^{(1)} E_p + \epsilon_0 \chi^{(3)} E_s^c E_s E_p$ . In the ideal case of purely radiative broadening of the excited states, and in the limit where the detunings are much larger than the linewidth ( $\Delta_p$ ,  $\Delta_s \gg \Gamma$ ), the conditional phase shift (CPS) and absorption of the probe beam is:

$$CPS \simeq n_s \left(\frac{3}{8\pi}\right) \left(\frac{\lambda_s^2}{A}\right) \left(\frac{1}{\tau \Delta_s}\right) \left(\frac{\Gamma}{\Delta_p}\right)^2 OD$$

$$Absorption \simeq \left(\frac{\Gamma}{\Delta_p}\right)^2 OD.$$
(2)

Here,  $n_s$  is the number of photons in the switch pulse and OD is the on-resonance optical depth.  $\lambda_s$  is the wavelength of the switch field, A is the spatial cross-sectional area of the two beams, and  $\tau$  is the pulse duration of the two beams (the two beams are assumed to have the same temporal and spatial characteristics). To avoid significant reshaping of the beams, we must choose the bandwidth of the beams to be much smaller when compared with the detunings,  $1/\tau \ll \Delta_p$ ,  $\Delta_s$ . From Eq. (2), for a high transmission of >50% and a CPS of  $\pi$  radians, a thousand photons per atomic cross-section is required.

The plot in Fig. 1 shows a numerical example based on Eq. (1). Here we use parameters that are typical for alkali atoms: wavelength  $\lambda = 780$  nm and decay rate  $\Gamma = 2\pi \times 6$  MHz. We take  $\Delta_p = \Delta_s = 160\Gamma$ ,  $\tau = 20$  ns, and assume the ideal case of  $A = \lambda^2/2\pi$  and take OD = 13,000. For these parameters, the transmission of the probe beam at the end of the medium is 60%. We find a CPS of  $\pi$  radians on the probe beam for a switch pulse containing 1000 photons per atomic cross-section. As we will discuss below, exact numerical simulations verify these results and demonstrate insignificant reshaping of the beams while propagating through the medium.

We proceed with a numerical study of the system. We neglect Doppler broadening and collisional effects and begin with a Hamiltonian describing a closed, three-level V-system in local time t' = t - z/c

$$\mathscr{H} = \hbar \begin{pmatrix} 0 & -\frac{\Omega_{p}(z,t')}{2} & -\frac{\Omega_{s}(z,t')}{2} \\ -\frac{\Omega_{p}(z,t')^{*}}{2} & \Delta_{p} & 0 \\ -\frac{\Omega_{s}(z,t')^{*}}{2} & 0 & \Delta_{s} \end{pmatrix}.$$
 (3)

We then use the commutator and anticommutator relations to find the equation of motion for the three-by-three density matrix  $\rho$  [1]:

$$\dot{\rho} = -\frac{i}{\hbar} [\mathscr{H}, \rho] - \frac{1}{2} \{\Gamma, \rho\}.$$
(4)

The values of  $\rho_{ij}$  calculated in Eq. (4) are used to numerically integrate the slowly varying envelope Maxwell's equations governing the propagation of the probe and switch fields,

$$\frac{\partial \Omega_{p}(z,t')}{\partial z} = -\frac{i}{\hbar} \eta \omega_{p} N \mu_{12}^{2} \rho_{12}(z,t')$$

$$\frac{\partial \Omega_{s}(z,t')}{\partial z} = -\frac{i}{\hbar} \eta \omega_{s} N \mu_{13}^{2} \rho_{13}(z,t'),$$
(5)

where  $\eta = \sqrt{\mu_0/\epsilon_0}$  is the impedance of free space. We solve Eqs. (4) and (5) with the initial condition that all atoms are in ground state  $|1\rangle$ . At the start of the atomic medium (z = 0) we apply a boundary condition that the fields, and therefore the Rabi frequencies  $\Omega_p(z = 0, t')$  and  $\Omega_s(z = 0, t')$ , are long Gaussian envelopes with a Gaussian width of  $\tau$ . Eqs. (4) and (5) are then solved on the space-time grid using the method of lines.

The results are presented in Fig. 2 and demonstrate a phaseshift of 3.2 radians with 60% transmission. In this simulation, we use the same parameters as the plot in Fig. 1 and use



**Fig. 2.** Numerical simulation of probe beam interacting with switch beam in an atomic medium. (a) The input probe pulse. (b) The resultant probe pulse in the case that the switch is off (blue, solid line) and on (red, dashed line). (c) The probe pulse phase accumulation with the switch off (blue, solid line) and with the switch on (red, dashed line), as a function of on-resonance optical depth. The switch pulse (not shown here) has a matching pulse-shape and frequency detuning. This means the pulse stay matched throughout the interaction. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



**Fig. 3.** The speed of the switch is increased at the expense of on-resonance optical depth. The detuning must be appropriately scaled to maintain low absorption. These plots assume an energy density of  $\sim$ 1000 photons per atomic cross-section.

 $n_s = 1000$  photons in the switch beam. We observe smooth timeprofiles at the end of the medium demonstrating negligible reshaping. Since the probe and the switch beam have identical detunings from the excited state, the switch pulse (not plotted) experiences near-identical absorption and reshaping. Furthermore, the two beams propagate with the same group velocity and therefore stay spatially and temporally well-matched while propagating through the medium.

Finally, we note that the energy, or number of switch photons, required for this phase-shifter is independent of bandwidth. Fig. 3 shows the required detuning and the density-length product for a given bandwidth that achieves the same performance as the numerical simulation of Fig. 2 (a CPS of  $\sim \pi$  radians for  $n_s = 1000$  switch photons). As the bandwidth broadens, both the probe and the switch must be appropriately detuned to avoid near-resonance effects. As noted in Eq. (2), if the increased switch detuning is accompanied by a shortened pulse duration, then the switch pulse is more intense for the same energy. The increased probe detuning trades off with an increased OD to keep the probe transmission constant.

We find that a proof-of-principle experiment showing the fast nature of this phase-shifter is within reach. A vapor cell experiment with  $N = 3 \times 10^{13}$  cm<sup>-3</sup> and a 30 µm beam waist would require a switch energy of around a picojoule ( $3 \times 10^7$  photons) to operate at a 50 MHz switching rate. These parameters achieve OD = 13,000, as quoted above, in 5 mm of optical length. Alternatively, an experiment in a hollow-core photonic band-gap fiber would be ideal. Exciting advances with this technology have lead to optical depths in excess of 1000 [14]. An optical depth of this magnitude confined to a 6 µm core diameter could result in a fast conditional phase-shifter to run on the  $3 \times 10^6$  photon level.

In summary, we suggested a far-off resonance scheme that supplies a conditional phase shift of  $\pi$  with an energy density of 1000 photons per  $\lambda^2/(2\pi)$ . To the best of our knowledge, the phase-shifter presented here is among the simplest of those suggested in the literature. As mentioned before, a possible application of our suggestion is to all-optical information processing. Among our future investigations is whether our approach may achieve switching at the single photon level, possibly with a high-finesse cavity. If the switch beam can be supplied by a single photon, then the suggestion described here may be applicable as a single-photon controlled-NOT gate between the probe and the switch. This will be among our future investigations.

### Acknowledgement

We thank J. T. Green for helpful discussions. This work was supported by Air Force Office of Scientific Research (AFOSR) and University of Wisconsin Alumni Research Foundation (WARF).

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