

Nanometer-scale optical traps using atomic state localization

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We suggest a scheme where a laser beam forms an optical trap with a spatial size that is much smaller than the wavelength of light. The key idea is to combine a far-off-resonant dipole trap with a scheme that localizes an atomic excitation.

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Over the last two decades, far-off-resonant optical dipole traps have evolved into one of the most widely used tools in trapping and manipulating ultracold neutral atoms [1,2]. By using a sufficiently large detuning from the excited electronic state, these traps can be made nearly dissipation free with photon-scattering rates as low as 1 Hz and with trap depths easily exceeding tens of millikelvins. Depending on the sign of the detuning from the excited electronic state, the atoms are trapped in either the intensity maxima or the minima of the optical field. Since the optical trapping potential is proportional to the intensity of the light, the trap size of a single focused beam is limited by the wavelength of light, λ . A counterpropagating beam pair can form an optical lattice with a spatial period of $\lambda/2$.

For a variety of tasks, optical traps with spatial scales smaller than the wavelength of light would be useful. As an example, traps at the nanometer scale may provide an important tool to control collisional interaction between two single atoms with nanometer resolution. Reliable control of such interactions may allow high-fidelity two-qubit gates between neutral atoms [3–6]. In this Brief Report, we suggest a scheme where a focused trapping beam can form an optical potential with a spatial size that is much smaller than the wavelength of light. The key idea is to combine a far-off-resonant dipole trap with a scheme that tightly localizes the atomic excitation to a specific state [7–14]. It is well known that by using the nonlinear interaction between atoms and laser beams, one can localize an atomic excitation to nanometer spatial scales. In their pioneering work, Thomas and co-workers [7,8] suggested and experimentally demonstrated subwavelength state localization of atoms using spatially varying energy shifts. Recently, there have been a number of suggestions that utilize manipulation of atoms at the subwavelength scale [9–14]. We propose to combine these ideas with a dipole trapping beam and suggest a simple scheme for obtaining nanometer-scale optical traps.

Before proceeding with a detailed description, we present a brief summary of our suggestion. Noting Fig. 1, we consider a three-level atomic system interacting with two laser beams, a trapping beam with a Rabi frequency Ω_S , and a localization beam with a Rabi frequency Ω_L . The localization laser couples the two lower metastable states of the atom, states $|1\rangle$ and $|2\rangle$. Throughout this Brief Report, we will assume the decay times of these two lower states to be much longer when compared with the other time scales of the problem. When $|1\rangle$ to $|2\rangle$ transition is dipole forbidden, the coupling can be produced by, for example, two Raman beams that are very far detuned from the excited states. For

this case, the quantity Ω_L is the effective two-photon Rabi frequency ($\Omega_L = \Omega_{r1}\Omega_{r2}/2\Delta_r$). If an atom is in state $|2\rangle$ and with the localization laser turned off ($\Omega_L = 0$), the far-off-resonant trapping beam forms a nearly conservative optical potential. This potential is given by the ac Stark shift of state $|2\rangle$, $U = \hbar|\Omega_S|^2/4\Delta_S$, and follows the intensity of the trapping laser. If the atom is initially in state $|1\rangle$ and with $\Omega_L \neq 0$, the situation is drastically different. Here, due to the ac Stark shift, the effective detuning δ_L (and therefore the excitation to state $|2\rangle$) becomes position dependent. In the limit where the magnitude of the Stark shift is much larger when compared with the localization laser Rabi frequency, the population of state $|2\rangle$ can be localized to spatial scales that are much smaller than the beam size of the trapping laser. Since the atom experiences the optical force due to the trapping laser only when state $|2\rangle$ has appreciable population, nanometer-scale optical traps can be constructed.

We proceed with the analysis of the system in Fig. 1. We make the Raman-Nath approximation and neglect the kinetic energy of the atoms when compared with the interaction energy with the fields, $\hbar^2k^2/2m \ll \hbar\Omega_L, \hbar\Omega_S$. If the detuning from the excited electronic state, Δ_S , is sufficiently large,

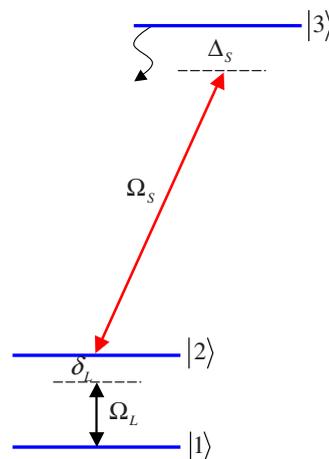


FIG. 1. (Color online) The scheme for constructing subwavelength optical traps using atomic state localization. The intense trapping beam with a Rabi frequency Ω_S forms a far-off-resonant dipole trap for the atom if the atom is in state $|2\rangle$. The weak localization laser with Rabi frequency Ω_L couples the two metastable states of the atom. With an atom starting in state $|1\rangle$, the excitation to state $|2\rangle$ is tightly localized due to position-dependent ac Stark shift of this state. As a result, dipole traps that are much smaller than the wavelength of light can be constructed.

dissipation due to spontaneous emission may be ignored. Within these assumptions, the following Hamiltonian describes the interaction of the fields with the atomic system:

$$\hat{H} = -\frac{\hbar}{2} \begin{bmatrix} 0 & \Omega_L & 0 \\ \Omega_L^* & -2\delta_L & \Omega_S \\ 0 & \Omega_S^* & -2\Delta_S \end{bmatrix}. \quad (1)$$

The detunings from the atomic transitions are defined as $\delta_L = (\omega_2 - \omega_1) - \omega_L$ and $\Delta_S = (\omega_3 - \omega_2) - \omega_S$, respectively. Throughout this Brief Report, for simplicity, we are going to consider only one spatial dimension. All of the arguments and the results can be extended to three spatial dimensions in a straightforward way. With the Hamiltonian of Eq. (1), the force on the atom can be found by $F = \langle \psi | -\frac{\partial \hat{H}}{\partial x} | \psi \rangle$ and is [1]

$$F = \frac{\hbar}{2} \left(\frac{\partial \Omega_L}{\partial x} \rho_{21} + \frac{\partial \Omega_S}{\partial x} \rho_{32} \right) + \text{c.c.} \quad (2)$$

Here, the quantities ρ_{21} and ρ_{32} are the off-diagonal density-matrix elements (coherences) between relevant transitions. With the average force on the atoms given by Eq. (2), the optical potential U can be found by using $U = -\int F dx$.

We proceed with a perturbative and steady-state solution of the coupled atom-light system. For this purpose, we take the Rabi frequency of the localization laser to be sufficiently small such that most of the population remains in the ground state, $\rho_{11} \approx 1$. This assumption requires $|\Omega_L| < |\delta_L - |\Omega_S|^2/4\Delta_S|$ at all spatial points. With this assumption, the values of the off-diagonal density-matrix elements at steady state are

$$\rho_{21} = \frac{\Omega_L^*}{2 \left(\delta_L - \frac{|\Omega_S|^2}{4\Delta_S} \right)},$$

$$\rho_{32} = \frac{|\Omega_L|^2 \Omega_S^*}{8 \left| \delta_L - \frac{|\Omega_S|^2}{4\Delta_S} \right|^2 \Delta_S}. \quad (3)$$

We have verified the validity of the solution of Eq. (3) by numerically solving the Bloch equations for the density-matrix elements. With the coherences of Eq. (3), we can calculate the force on the atom and therefore estimate the optical potential. Before proceeding with the numerical results, we first present an analytical estimate. For this purpose, we take the localization laser to be uniform and assume Gaussian focusing for the trapping laser beam with a spot size of W_S , $\Omega_S(x) = \Omega_{S0} \exp[-x^2/W_S^2]$. We consider the tight-localization limit where the Rabi frequency of the localization laser is much weaker when compared with the peak Stark shift due to the trapping beam, $\Omega_L \ll \Omega_{S0}^2/4\Delta_S$. In this limit, if the minimum value of the Stark-shifted detuning, $\delta_L - \Omega_{S0}^2/4\Delta_S$, is of order Ω_L , it can be derived from Eq. (3) that the optical potential will have a spot size of $\approx W_S \sqrt{\Omega_L/(\Omega_{S0}^2/4\Delta_S)}$ with a trap depth on the order of $\hbar\Omega_L$. For a fixed trapping laser beam parameters, by choosing the ratio $\Omega_L/(\Omega_{S0}^2/4\Delta_S)$ to be small, the trap size can be made

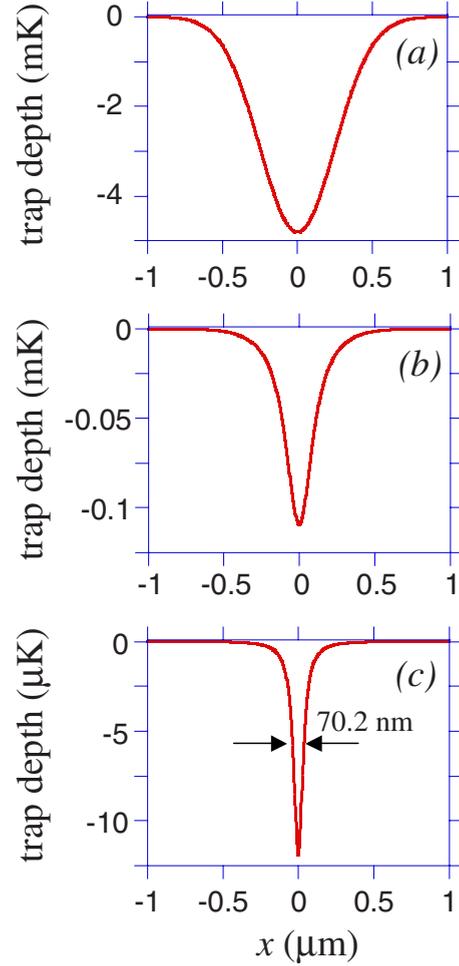


FIG. 2. (Color online) (a) The dipole potential seen by the atom when the atom is in state $|2\rangle$ and $\Omega_L = 0$ (without any localization). The trapping beam has a Gaussian spot size of $W_S = 0.5 \mu\text{m}$ and induces a peak Stark shift of $\Omega_{S0}^2/4\Delta_S = 2\pi \times 100$ MHz. A trap with a size of about $1 \mu\text{m}$ and a depth of about 4.5 mK is formed. (b) The dipole potential for an atom that starts in state $|1\rangle$ with a localization laser Rabi frequency of $\Omega_L = 2\pi \times 10$ MHz. (c) The dipole potential with a localization laser Rabi frequency of $\Omega_L = 2\pi \times 1$ MHz. The trap has a size of 70.2 nm with a depth of $11.9 \mu\text{K}$. The trapping laser parameters are identical for all three cases. As the localization laser Rabi frequency is decreased, the trap size decreases at the expense of a reduced trap depth.

arbitrarily smaller than the wavelength of light at the expense of a reduced trap depth.

We proceed with a numerical example. We choose the trapping beam intensity and the detuning to give a peak Stark shift of $\Omega_{S0}^2/4\Delta_S = 2\pi \times 100$ MHz. We assume a focusing spot size of $W_S = 0.5 \mu\text{m}$ and, for simplicity, take the localization laser to be uniform. Figure 2(a) shows the dipole potential seen by the atom when the atom is in state $|2\rangle$ and $\Omega_L = 0$ (without any localization). A trap that is proportional to the intensity of the trapping laser beam with a depth of about 4.5 mK is formed. Figure 2(b) is the dipole potential if the atom starts in state $|1\rangle$ with a localization laser Rabi frequency of $\Omega_L = 2\pi \times 10$ MHz. Here, we choose the detuning δ_L such that the peak fractional population of state $|2\rangle$ is

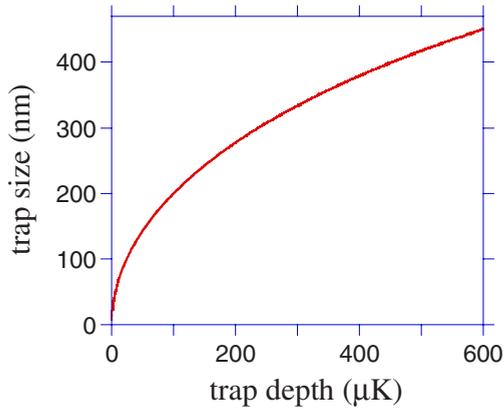


FIG. 3. (Color online) The trap size and the trap depth of the optical potential as the localization laser Rabi frequency is varied. The trapping beam parameters are identical to those of Fig. 2 with a peak stark shift $\Omega_{S0}^2/4\Delta_S = 2\pi \times 100$ MHz and a Gaussian spot size of $W_S = 0.5 \mu\text{m}$. As the localization laser Rabi frequency is reduced, the size of the trap becomes smaller at the expense of a decrease in the trap depth.

0.25. In Fig. 4(c), we reduce the localization laser Rabi frequency to $\Omega_L = 2\pi \times 1$ MHz. The detuning δ_L is again adjusted to give a peak fractional population of 0.25. A trap with a full width at half maximum (FWHM) size of 70.2 nm and a depth of 11.9 μK is formed. The trapping laser parameters are identical for all the three plots of Fig. 2.

Figure 3 shows the trap size as a function of the trap depth while the localization laser Rabi frequency Ω_L is varied. Here, we use the same trapping beam parameters as in Fig. 2 and adjust the detuning δ_L such that the peak fractional population of state $|2\rangle$ is 0.25 at all points in the plot. The decrease in the trap size comes at the expense of a reduced trap depth. As the localization laser Rabi frequency Ω_L becomes small, the validity of Raman-Nath approximation requires correspondingly smaller atomic temperatures.

One key feature of traps with nanometer spatial scales is the decreased number of vibrational motional states in the trap. Figure 4 demonstrates this feature. Here, we consider a ^{23}Na atom trapped in the external trapping potential of Fig. 2(c) (a trap size of 70.2 nm and a trap depth of 11.9 μK). We numerically calculate the bound-state eigenvalues and eigenfunctions by using the Fourier grid Hamiltonian method [15]. We find two bound states, $|\nu=0\rangle$ and $|\nu=1\rangle$, with confinement energies of 5.79 and 0.3 μK , respectively. The plots show the numerically calculated probability distributions for the two eigenstates. The eigenfunction for state $|\nu=0\rangle$ is localized to a FWHM size of 75.6 nm. Having a small number of bound states may be advantageous for a number of reasons. As an example, optical traps with a single bound state do not require cooling inside the trap.

We envision that the first experimental demonstration of our suggestion will likely use alkali-metal atoms that are trapped and cooled inside a magneto-optical trap (MOT). In alkali-metal atoms, the stretched states of the hyperfine levels may be used as the two metastable states required by the scheme. As an example, for ^{23}Na , these states can be chosen as $|1\rangle \rightarrow |F=2, m_F=2\rangle$ and $|2\rangle \rightarrow |F=1, m_F=1\rangle$. A right-hand circularly polarized trapping laser beam (σ^+) may then be

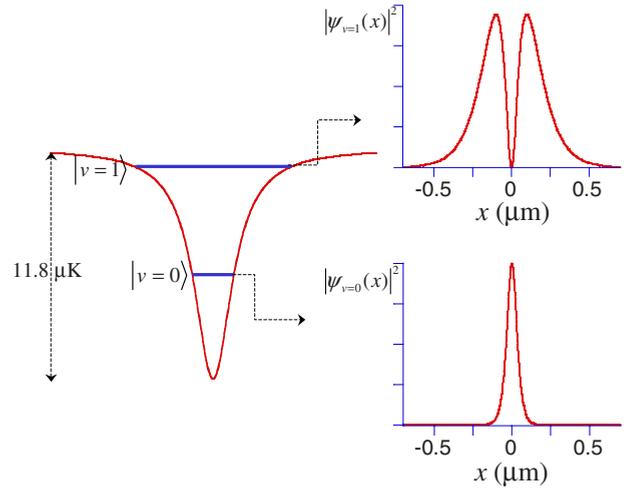


FIG. 4. (Color online) The bound states of a ^{23}Na atom that is trapped in the external potential of Fig. 2(c). We numerically calculate the eigenvalues and eigenfunctions of the Schrödinger equation and find two bound states, $|\nu=0\rangle$ and $|\nu=1\rangle$, with confinement energies of 5.79 and 0.3 μK , respectively. The plots show the probability distributions for the eigenstates, $|\psi_{\nu=0}(x)|^2$ and $|\psi_{\nu=1}(x)|^2$.

used to couple state $|2\rangle$ to the excited state $|3\rangle \rightarrow |F'=2, m_{F'}=2\rangle$ in the D_1 line. Due to angular momentum selection rules, this configuration suppresses an undesired coupling of states $|1\rangle$ and $|3\rangle$ with the trapping laser beam. The suppression of coupling requires negligible interaction with the D_2 line, which in turn limits the largest value for the detuning from the D_1 line, Δ_S , that can be used for the trapping beam. This detuning must be small when compared with the separation of the D_1 and D_2 lines. For ^{23}Na , this imposes a limit of $\Delta_S \approx 100$ GHz, which results in a spontaneous scattering rate of about 10 kHz from the trapping beam for the numerical simulations of Figs. 2–4. This limitation can be overcome by using an atomic species with a larger energy separation of the D_1 and D_2 lines. As an example, for ^{87}Rb , the separation of the two lines is 7.1 THz, and as a result, the undesired photon-scattering rate can be kept well below 1 kHz.

We also note that the required laser parameters for the experimental observation of our suggestion with alkali-metal atoms are modest. Typically, a laser power of only tens of milliwatts in the trapping beam is required. To avoid fluctuations in the ac Stark shift, the intensity stability of the trapping laser beam is critical. The fractional intensity fluctuations in the trapping laser must be small when compared with the ratio $\Omega_L/(\Omega_{S0}^2/4\Delta_S)$. The results of Fig. 2(c) would, therefore, require an intensity stability better than 1%. The localization coupling rate Ω_L between the lower hyperfine states may be produced by two laser beams whose frequency difference equals the hyperfine transition frequency (1.77 GHz in ^{23}Na). These two lasers can be very far detuned from the D lines to keep a sufficiently low photon-scattering rate.

In conclusion, we have suggested a scheme for generating nanometer-scale optical traps. As mentioned before, one

exciting application of our scheme is to quantum computing and quantum information processing. Due to the extremely small trapping volume, the collisional blockade mechanism of Grangier and co-workers [16,17] will be very pronounced while loading our trap from an ultracold gas background. As a result, only one atom may be loaded to the trap with a very high fidelity. This may produce a deterministic source of single atoms, or in a lattice geometry, may allow determin-

istic loading and initialization of a neutral atom quantum register [18].

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- [1] H. J. Metcalf and P. van der Straten, *Laser Cooling and Trapping* (Springer-Verlag, New York, 1999).
- [2] J. D. Miller, R. A. Cline, and D. J. Heinzen, *Phys. Rev. A* **47**, R4567 (1993).
- [3] G. K. Brennen, C. M. Caves, P. S. Jessen, and I. H. Deutsch, *Phys. Rev. Lett.* **82**, 1060 (1999).
- [4] D. Jaksch, H. J. Briegel, J. I. Cirac, C. W. Gardiner, and P. Zoller, *Phys. Rev. Lett.* **82**, 1975 (1999).
- [5] P. Cheinet, S. Trotzky, M. Feld, U. Schnorrberger, M. Moreno-Cardoner, S. Fölling, and I. Bloch, *Phys. Rev. Lett.* **101**, 090404 (2008).
- [6] M. Anderlini, P. J. Lee, B. L. Brown, J. Sebby-Strabley, W. D. Phillips, and J. V. Porto, *Nature (London)* **448**, 452 (2007).
- [7] K. D. Stokes, C. Schnurr, J. R. Gardner, M. Marable, G. R. Welch, and J. E. Thomas, *Phys. Rev. Lett.* **67**, 1997 (1991).
- [8] J. R. Gardner, M. L. Marable, G. R. Welch, and J. E. Thomas, *Phys. Rev. Lett.* **70**, 3404 (1993).
- [9] F. Le Kien, G. Rempe, W. P. Schleich, and M. S. Zubairy, *Phys. Rev. A* **56**, 2972 (1997).
- [10] S. Qamar, S. Y. Zhu, and M. S. Zubairy, *Phys. Rev. A* **61**, 063806 (2000).
- [11] E. Paspalakis and P. L. Knight, *Phys. Rev. A* **63**, 065802 (2001).
- [12] G. S. Agarwal and K. T. Kapale, *J. Phys. B* **39**, 3437 (2006).
- [13] D. D. Yavuz and N. A. Proite, *Phys. Rev. A* **76**, 041802(R) (2007).
- [14] A. V. Gorshkov, L. Jiang, M. Greiner, P. Zoller, and M. D. Lukin, *Phys. Rev. Lett.* **100**, 093005 (2008).
- [15] C. C. Marston and G. G. Balint-Kurti, *J. Chem. Phys.* **91**, 3571 (1989).
- [16] N. Schlosser, G. Reymond, I. Protsenko, and P. Grangier, *Nature (London)* **411**, 1024 (2001).
- [17] N. Schlosser, G. Reymond, and P. Grangier, *Phys. Rev. Lett.* **89**, 023005 (2002).
- [18] D. Schrader, I. Dotsenko, M. Khudaverdyan, Y. Miroshnychenko, A. Rauschenbeutel, and D. Meschede, *Phys. Rev. Lett.* **93**, 150501 (2004).