



Elimination of Feshbach loss in a Bose–Einstein condensate

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Abstract

We suggest a technique to eliminate inelastic losses in an atomic condensate when tuned close to a Feshbach resonance. The key idea is to couple the quasi-bound molecular state to a bound molecular state with an electromagnetic field. Such coupling forces the population of the Feshbach state to zero, thereby eliminating all of the losses associated with this state.

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In recent experiments on Bose–Einstein condensates large loss rates were observed when a quasi-bound molecular state was tuned slowly close to a Feshbach resonance with an atomic condensate [1,2]. Although the precise mechanism for this loss is not understood, it is likely that the loss is associated with the population of the quasi-bound molecular state. As suggested by Yurovsky et al. [3], one mechanism which causes loss is three body recombination.

In this work we extend the suggestion of Harris [4] to coupled, zero-temperature, atomic and molecular condensates, and demonstrate a technique to eliminate Feshbach loss. The key idea is to

couple the quasi-bound molecular state to a bound molecular state with an electromagnetic field. Due to destructive interference, the quasi-bound molecular state accumulates no population and all of the losses associated with this state are eliminated. The nature of this interference has strong similarities with electromagnetically induced transparency (EIT) for light fields, where an incident optical beam tuned to what was previously line center has near zero absorption [5].

A schematic of the system to be studied is shown in Fig. 1. The energy of the atomic condensate is magnetically tuned close to a Feshbach resonance with the quasi-bound diatomic molecules. We assume that the quasi-bound molecules decay with a rate Γ_1 . The quantity Ω in Fig. 1 is the Rabi frequency of the electromagnetic field that couples the quasi-bound and bound molecular states. In symmetric molecules this transition is dipole forbidden and Ω is the effective two-photon

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$\hbar\omega_1 = V_1 + \sum_i g_{1i} |\phi_i(0)|^2$, and $\hbar\omega_2 = V_2 + \sum_i g_{2i} |\phi_i(0)|^2$; $\Delta\omega = (2\omega_a - \omega_1)$ is the detuning from the Feshbach resonance. With these definitions, the coupled equations are:

$$i \frac{\partial \phi_a}{\partial t} = 2\alpha^* \phi_1 \phi_a^* + \frac{1}{\hbar} \left[\sum_i g_{ai} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_a, \quad (2a)$$

$$\begin{aligned} \left(\Delta\omega + i \frac{\Gamma_1}{2} \right) \phi_1 + i \frac{\partial \phi_1}{\partial t} \\ = \alpha \phi_a^2 + \frac{\Omega}{2} \phi_2 + \frac{1}{\hbar} \left[\sum_i g_{1i} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_1, \end{aligned} \quad (2b)$$

$$\begin{aligned} \left(\Delta\omega + i \frac{\Gamma_2}{2} \right) \phi_2 + i \frac{\partial \phi_2}{\partial t} \\ = \frac{\Omega^*}{2} \phi_1 + \frac{1}{\hbar} \left[\sum_i g_{2i} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_2. \end{aligned} \quad (2c)$$

In the above equations, we have taken the coupling field to be on exact resonance with the molecular states and used the rotating-wave approximation. For simplicity, we have taken Γ_1 to be a constant, and neglected the density dependent decay of the atomic and molecular condensates due to atom–molecule and molecule–molecule collisions. The inclusion of these decay terms do not change our results significantly. From Eqs. (2), with $\Gamma_1 = \Gamma_2 = 0$, we obtain the conservation condition for the total atom number: $\frac{\partial}{\partial t} (|\phi_a|^2 + 2|\phi_1|^2 + 2|\phi_2|^2) = 0$.

We proceed by numerically solving Eqs. (2) with the parameters of an MIT experiment [2,3]. We consider the 907 G Feshbach resonance in a Na atomic condensate with $|\phi_a(0)|^2 = 5 \times 10^{14} / \text{cm}^3$, $a_{aa} = 3.4$ nm, and $\alpha = 4.11 \times 10^{-5}$ (mks). We take $\Gamma_2 = 0.01$ Hz [11], consider an on-resonance excitation ($\Delta\omega = 0$), and take $\Gamma_1/2\pi = 0.3$ MHz. The choice of Γ_1 is consistent within an order of magnitude to the estimated rates of van Abeleen et al. [9], Yurovsky et al. [3] and Timmermans et al. [19]. We also assume $\phi_1(0) = \phi_2(0) = 0$. Although the atom–molecule and molecule–molecule interaction constants are not known, our results are

insensitive to their exact values. In Fig. 2, the normalized density of the atomic condensate, $|\phi_a|^2$, is plotted vs. time for $\Omega = 0$ and $\Omega/2\pi = 1$ MHz, respectively. Without the coupling field, the condensate decays very rapidly; the loss is due to the formation of the quasi-bound molecules within the condensate, followed by the inelastic decay of these molecules. With the coupling field on, due to destructive interference, the loss to the atomic condensate is eliminated. The initial decrease in the density is due to the production of bound molecules, which is essential for creating interference. This phenomenon is in the spirit of the preparation energy for EIT with light fields [21].

The last terms on the right-hand side of Eqs. (2) are a density-dependent frequency shift of the mean-field amplitudes. We now proceed with the analytical interpretations of the numerical results of Fig. 2 and neglect the frequency shifts in Eqs. (2). We assume $\frac{\partial \phi_1}{\partial t} \ll \Gamma_1 \phi_1$ and $\frac{\partial \phi_2}{\partial t} \ll \frac{|\Omega|^2}{2\Gamma_1} \phi_2$ in Eqs. (2b) and (2c) to obtain expressions for the molecular field amplitudes:

$$\phi_1 = \frac{2\alpha(2\Delta\omega + i\Gamma_2)}{(2\Delta\omega + i\Gamma_1)(2\Delta\omega + i\Gamma_2) - |\Omega|^2} \phi_a^2, \quad (3a)$$

$$\phi_2 = \frac{2\alpha\Omega^*}{(2\Delta\omega + i\Gamma_1)(2\Delta\omega + i\Gamma_2) - |\Omega|^2} \phi_a^2. \quad (3b)$$

With these expressions, the differential equation for the atomic mean field amplitude (Eq. (2a)) is:

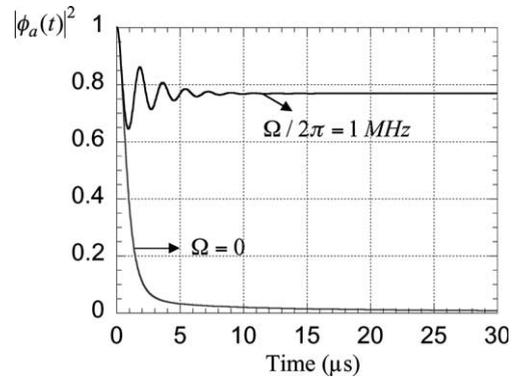


Fig. 2. The normalized density of the atomic condensate, $|\phi_a|^2$, vs. time for $\Omega = 0$ and $\Omega/2\pi = 1$ MHz. With the coupling field on, the decay of the atomic condensate is eliminated.

$$i \frac{\partial \phi_a}{\partial t} = \frac{4|\alpha|^2(2\Delta\omega + i\Gamma_2)}{(2\Delta\omega + i\Gamma_1)(2\Delta\omega + i\Gamma_2) - |\Omega|^2} |\phi_a|^2 \phi_a$$

$$\equiv \frac{4\pi\hbar}{m} a(\Delta\omega) |\phi_a|^2 \phi_a, \quad (4)$$

where $a(\Delta\omega)$ is the complex scattering length for the atomic condensate (without the background contribution). The imaginary part of $a(\Delta\omega)$ determines the loss to the condensate and is plotted vs. frequency in Fig. 3(a) for the parameters of the numerical simulation of Fig. 2. With the coupling field, the loss profile is double-peaked and is zero on the exact Feshbach resonance (previous maximum). By choosing the intensity of the coupling field, the interference profile may be either narrower or wider than the decay width, Γ_1 . The real part of $a(\Delta\omega)$ is the change in the atom–atom interaction energy due to the Feshbach interaction and is plotted versus frequency in Fig. 3(b) [22].

As seen from Fig. 3(a), perfect transparency is only obtained on exact Feshbach resonance. Due to the nonlinear frequency shifts of Eqs. (2), an

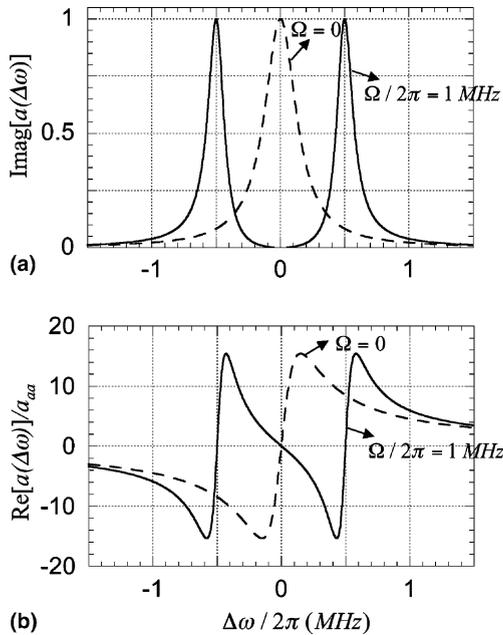


Fig. 3. (a) The normalized imaginary part of the scattering length. With the coupling field on, there is no loss on exact Feshbach resonance. (b) The real part of the scattering length normalized to its background value.

atomic condensate which is initially tuned to the exact Feshbach resonance will go in and out of the resonance during coherent interactions and the transparency will be degraded. One way to eliminate these shifts is to bound the minimum intensity of the coupling field thereby limiting the number of generated molecules. From Eqs. (3), this minimum intensity bound is:

$$|\Omega|^2 \gg 4|\alpha|^2 |\phi_a|^2. \quad (5)$$

This restriction might be overcome by tracking the resonance using a time varying magnetic field or a frequency-chirped coupling laser.

As noted above, throughout this paper, for simplicity, we have taken Γ_1 to be constant and neglected density dependent decay of the atomic and molecular condensates due to atom–molecule collisions. With the inclusion of these decay terms, Eqs. (2) read:

$$i \frac{\partial \phi_a}{\partial t} + i\gamma |\phi_1|^2 \phi_a$$

$$= 2\alpha^* \phi_1 \phi_a^* + \frac{1}{\hbar} \left[\sum_i g_{ai} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_a,$$

$$\left(\Delta\omega + i \frac{\Gamma_1}{2} + i\gamma |\phi_a|^2 \right) \phi_1 + i \frac{\partial \phi_1}{\partial t}$$

$$= \alpha \phi_a^2 + \frac{\Omega}{2} \phi_2 + \frac{1}{\hbar} \left[\sum_i g_{1i} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_1,$$

$$\left(\Delta\omega + i \frac{\Gamma_2}{2} \right) \phi_2 + i \frac{\partial \phi_2}{\partial t}$$

$$= \frac{\Omega^*}{2} \phi_1 + \frac{1}{\hbar} \left[\sum_i g_{2i} (|\phi_i(t)|^2 - |\phi_i(0)|^2) \right] \phi_2, \quad (6)$$

where γ is the atom–molecule state changing collision rate [3,19]. In Eq. (6), Γ_1 represents any other unknown loss mechanism of the fragile quasi-bound state. Numerically solving Eq. (6), with $\gamma \approx 10^{-9}–10^{-10}$ cm³/s [3,19], and with all the other parameters identical, we find that the results of Fig. 2 practically remain unchanged.

In our formalism, for simplicity, we have not included an upper electronic molecular state, which is necessary for two-photon coupling the quasi-bound and bound molecular states in symmetric molecules. For the numerical simulations of

Fig. 2, the inclusion of such a state doesn't change the results significantly as long as the detuning from this state is larger than 10 times the decay width of this state. Strong Raman coupling for these conditions require laser power densities of about 10 kW/cm². Tuning closer to the upper electronic state reduces the laser intensity requirement, however degrades transparency. The amount of degradation can be found by adiabatically eliminating the mean-field amplitude of the upper electronic state and substituting the relevant quantities in Eq. (4) with their effective values. These substitutions are: $\Omega \rightarrow \Omega_1\Omega_2^*/(2\delta\omega + i\Gamma_3)$, $\Gamma_1 \rightarrow \Gamma_1 - \text{Imag}[\Omega_1^2/(2\delta\omega + i\Gamma_3)]$, and $\Gamma_2 \rightarrow \Gamma_2 - \text{Imag}[\Omega_2^2/(2\delta\omega + i\Gamma_3)]$. Here Ω_1 and Ω_2 are the Rabi frequencies of the two driving lasers, $\delta\omega$ is the detuning from upper state, and Γ_3 is the decay width of this state.

Several recent papers discuss the importance of the noncondensate modes in the coupled condensate dynamics [14,23–25]. We believe that the inclusion of these modes will not change the results of this letter significantly since the quasi-bound state does not accumulate significant population.

In summary, we have suggested a technique to eliminate losses to an atomic condensate near a Feshbach resonance. The predictions of this letter can be realized within current experimental conditions. By coupling the quasi-bound state, the double-peaked interference profile of Fig. 3(a) should be readily observable. The atomic condensate experiences dispersion of the scattering length at the point of zero loss. Such a unique dispersive feature may find possible applications in nonlinear matter–wave interactions.

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