Single photon SWAP gate using electromagnetically induced transparency

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We describe a scheme that performs a SWAP gate between two photons at different wavelengths with near 100% fidelity. The essential idea is the preparation of a near-maximal atomic coherence using electromagnetically induced transparency.

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Over the last decade quantum computation has received much attention due to the possibility of solving certain problems more efficiently than a classical computer [1]. Currently a number of different approaches are being pursued to build a scalable quantum computer. An attractive physical system for implementing quantum computation uses photons at different wavelengths as qubits [1]. Photons are ideal carriers of quantum information since they suffer little from decoherence. However, it is a real challenge to interact single photons, since the nonlinearities in typical materials are very small.

Resonantly enhanced nonlinearities using electromagnetically induced transparency (EIT) show considerable promise for interacting single photons at different wavelengths [2–7]. These proposals utilize unique dispersive properties of EIT to obtain unusually large cross phase or amplitude modulation. One common theme of these proposals is the slow light associated with the steep dispersion of the refractive index. The limitations of these proposals are as follows: (1) The nonlinear effects are proportional to the intensity of light. As a result very tight focusing is necessary to observe significant interactions at the single photon level. (2) Since slow light is essential, the quantum gates that are constructed are necessarily slow.

In this paper, we suggest a scheme that performs a SWAP gate between two photons at different wavelengths. Our proposal does not suffer from the limitations of the previous paragraph. However, a SWAP gate alone is not sufficient to build a scalable quantum computer [8]. Therefore, our scheme should be thought as supplementing other proposals [2–7] in building an optical quantum computer. The scheme that will be analyzed in detail is shown in Fig. 1. We consider a four-level atomic system interacting with four fields. Two of these fields (termed the probe field E_p and the coupling field E_c) are strong and form a traditional EIT-lambda scheme. These fields drive the atoms to a dark state and prepare the coherence (off-diagonal density matrix element) of the nonallowed Raman ($|1\rangle$ to $|2\rangle$) transition. Two weak fields (termed E_a and E_b) then interact through the established coherence of the atomic system. When the coherence is near its maximum value $|\rho_{12}| \approx 0.5$, the propagation dynamics of E_a and E_b are strongly coupled to one another. As will be demonstrated below, under certain conditions, the interaction between E_a and E_b is identical to a conventional beamsplitter between two spatial modes. The rotation angle of the beamsplitter is set by the density length product of the atomic medium. When the rotation angle is $\pi/2$, the medium transfers the quantum state of E_a (E_b) to E_b (E_a).

Before proceeding further, we would like to cite pertinent earlier work: Several experiments have demonstrated nonlinear processes at low light levels using EIT [9–12]. By using the schematic of Fig. 1, Jain *et al.* and Merriam *et al.* have demonstrated near 100% conversion efficiencies in lead vapor [13,14]. Sokolov and colleagues have extended the ideas of maximum coherence to molecular systems and have demonstrated a very broad comb of Raman sidebands [15,16]. The quantum dynamics of four-wave mixing with EIT has been studied in detail by Fleischhauer and colleagues [17]. Resch *et al.* have demonstrated significant nonlinear effects at the single photon levels using spontaneous parametric down conversion [18].

We now proceed with a detailed description of our scheme. We are going to assume that E_p and E_c are sufficiently intense such that they can be treated classically. For the weak fields E_a and E_b, we consider the dynamics of the photon annihilation operators $\hat{a}_a$ and $\hat{a}_b$, respectively. We assume slowly varying envelopes when compared with the optical k vector. We make the rotating wave approximation and neglect the contribution of any other atomic levels to the propagation constant. With these assumptions, the coupled equations for the annihilation operators are

$$\frac{\partial \hat{a}_a}{\partial z} = -j\beta_a \rho_{11} \hat{a}_a - j\gamma_a \rho_{12} \hat{a}_b,$$

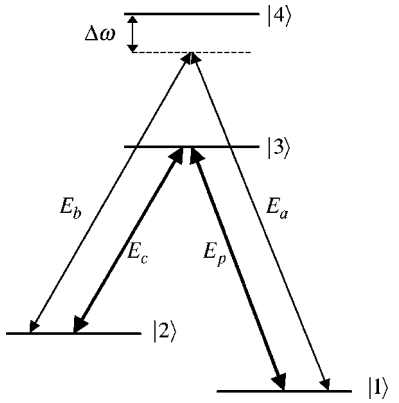

FIG. 1. The energy level diagram for the proposed scheme. Two strong fields E_p and E_c adiabatically drive the atoms to a maximally coherent state. The two weak fields E_a and E_b then influence each other through the established coherence.

$$\frac{\partial \hat{a}_b}{\partial z} = -j\gamma_b \rho_{12}^* \hat{a}_a - j\beta_b \rho_{22} \hat{a}_b. \tag{1}$$

Here ρ_{ij} are the density matrix elements of the $|1\rangle$ to $|2\rangle$ Raman transition. The weak fields are coupled to each other through ρ_{12}, which is the off-diagonal density matrix element. The constants β_a, β_b, γ_a, and γ_b determine dispersion and coupling and they are

$$\beta_a = \frac{1}{2}\eta\omega_a N \frac{|\mu_{14}|^2}{\hbar\Delta\omega}, \quad \beta_b = \frac{1}{2}\eta\omega_b N \frac{|\mu_{24}|^2}{\hbar\Delta\omega},$$

$$\gamma_a = \frac{1}{2}\eta\omega_a N \frac{\mu_{14}^* \mu_{24}}{\hbar\Delta\omega}, \quad \gamma_b = \frac{1}{2}\eta\omega_b N \frac{\mu_{14}\mu_{24}^*}{\hbar\Delta\omega}, \tag{2}$$

with N being the atom number density and $\eta=(\mu/\epsilon_0)^{1/2}$. μ_{ij} are the dipole matrix elements between respective transitions and $\Delta\omega$ is the common detuning of both fields from state $|4\rangle$ (we assume exact two photon resonance). Equation (1) can be derived by first writing the semiclassical slowly varying envelope propagation equation for E_a and E_b, and making the substitutions $E_a \rightarrow \hat{a}_a$, and $E_b \rightarrow \hat{a}_b$. Alternatively, these equations can also be derived by assuming a Hamiltonian of the form $\hat{H}=\hbar c(\beta_a\rho_{11}\hat{a}_a^\dagger\hat{a}_a+\gamma_a\rho_{12}\hat{a}_b^\dagger\hat{a}_a+\beta_b\rho_{22}\hat{a}_b^\dagger\hat{a}_b+\gamma_b\rho_{12}^*\hat{a}_a^\dagger\hat{a}_b)$, and then converting the Heisenberg equations of motion for the operators $\hat{a}_a$ and $\hat{a}_b$ into spatial differential equations with the change of variable $z=ct$ [19]. We assume $\Delta\omega$ to be sufficiently large such that dissipation can be neglected and drop the noise operators that would otherwise be on the right-hand side of Eq. (1).

The density matrix elements in Eq. (1) are driven by the strong fields E_c and E_p. With all atoms starting from the ground state $|1\rangle$, the atomic system can be prepared adiabatically with the system remaining in the dark state at all times. This is achieved by counterintuitive pulse sequence, i.e., turning the coupling field E_c on before the probe field E_p [20]. With the atoms in the dark state, the population of state $|3\rangle$ is zero, and the density matrix elements of the Raman transition are given by [13,14,20]

$$\rho_{11} = \frac{|\Omega_c|^2}{|\Omega_p|^2+|\Omega_c|^2}, \quad \rho_{22} = \frac{|\Omega_p|^2}{|\Omega_p|^2+|\Omega_c|^2},$$

$$\rho_{12} = -\frac{\Omega_p\Omega_c^*}{|\Omega_p|^2+|\Omega_c|^2}, \tag{3}$$

where Ω_p and Ω_c are the complex Rabi frequencies for the coupling and probe beams and are defined as $\Omega_p=E_p\mu_{13}/\hbar$, $\Omega_c=E_c\mu_{23}/\hbar$.

The adiabatic solution for the density matrix [Eq. (3)] and the propagation equations for the annihilation operators [Eq. (1)] completely describe the field-atom interaction. We now proceed with the analysis of these equations. To first order, the density matrix elements that appear in Eq. (1) are independent of distance, i.e., $\rho_{ij}(z)=\rho_{ij}(0)$. This is because, when the medium is in the dark state, the probe field, E_p, and the coupling field, E_c, propagate as in vacuum (i.e., without any amplitude and phase change) through the EIT medium. With this assumption, the coupled equations for the annihilation operators [Eq. (1)] are linear and their solution is

$$\begin{pmatrix} \hat{a}_a(z) \\ \hat{a}_b(z) \end{pmatrix} = \exp(-j\bar{\bar{M}}z)\begin{pmatrix} \hat{a}_a(0) \\ \hat{a}_b(0) \end{pmatrix} \equiv \begin{pmatrix} A(z) & B(z) \\ C(z) & D(z) \end{pmatrix}\begin{pmatrix} \hat{a}_a(0) \\ \hat{a}_b(0) \end{pmatrix}, \tag{4}$$

where

$$\bar{\bar{M}} = \begin{pmatrix} \beta_a\rho_{11}(0) & \gamma_a\rho_{12}(0) \\ \gamma_b\rho_{12}^*(0) & \beta_b\rho_{22}(0) \end{pmatrix}. \tag{5}$$

To further simplify the system we take the dispersive and coupling constants in the above to be the same: $\beta_a \approx \beta_b \approx \gamma_a \approx \gamma_b \equiv \beta$. For a state of maximum coherence, i.e., $|\rho_{12}| = \rho_{11} = \rho_{22} = \frac{1}{2}$, the matrix elements of $\exp(-j\bar{\bar{M}}z)$ that appear in Eq. (4) are

$$\begin{pmatrix} A(z) & B(z) \\ C(z) & D(z) \end{pmatrix} = \exp(-j\beta z/2)\begin{pmatrix} \cos(\beta z/2) & -j\exp(j\phi)\sin(\beta z/2) \\ -j\exp(-j\phi)\sin(\beta z/2) & \cos(\beta z/2) \end{pmatrix}. \tag{6}$$

Here we have defined $\rho_{12}=|\rho_{12}|\exp(j\phi)$. Equation (6) is analogous to a conventional beam splitter between two spatial modes with a rotation angle of $\beta z/2$ [1].

We now consider the evolution of the input states for the weak fields while propagating through the coherent EIT medium. We define the photon number operators as $\hat{n}_a(z)=\hat{a}_a^\dagger\hat{a}_a$ and $\hat{n}_b(z)=\hat{a}_b^\dagger\hat{a}_b$. We first consider the case where at the beginning of the EIT medium ($z=0$), each field is in a photon number eigenstate (Fock state), $|\psi\rangle_{input}=|n\rangle_a|m\rangle_b=|n,m\rangle$. The average number of photons in each field while propagating through the EIT medium and the fluctuations around this average are then $\langle\hat{n}_a(z)\rangle=|A(z)|^2n+|B(z)|^2m$, $\langle\Delta\hat{n}_a^2(z)\rangle=|A(z)|^2|B(z)|^2(n+m)$, $\langle\hat{n}_b(z)\rangle=|C(z)|^2n+|D(z)|^2m$, and $\langle\Delta\hat{n}_b^2(z)\rangle=|C(z)|^2|D(z)|^2(n+m)$. Using Eq. (6), if the cell length L is chosen such that $\beta L/2=\pi/2$, at the end of the cell the output state of the system will be $|\psi\rangle_{output}=(-1)^{n+m}\exp[j(m-n)\phi]|m,n\rangle$ [21]. The system, therefore, swaps the number of photons in each field up to a global phase factor. For the particular case when there is either 0 or 1 photon in each field, we have the following truth table for the input-output states of the fields:

$$|0,0\rangle \rightarrow |0,0\rangle,$$

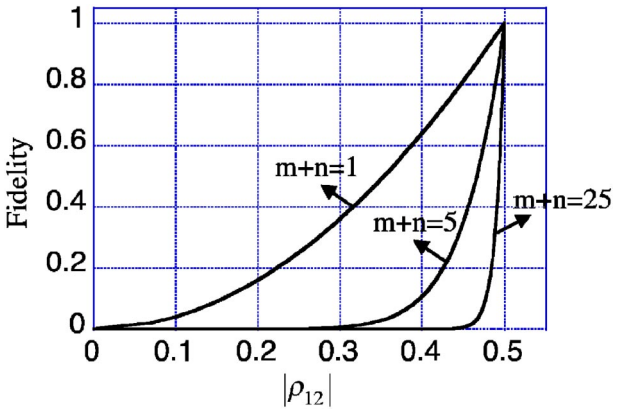

FIG. 2. The fidelity, F, of the SWAP gate as a function of the magnitude of the atomic coherence, $|\rho_{12}|$. For a maximally coherent atomic medium, the fidelity of the SWAP operation is unity.

$$|0,1\rangle \rightarrow -\exp(j\phi)|1,0\rangle,$$

$$|1,0\rangle \rightarrow -\exp(-j\phi)|0,1\rangle,$$

$$|1,1\rangle \rightarrow |1,1\rangle. \tag{7}$$

This truth table constitutes a SWAP gate for single photons. A photon in field E_a (E_b) is converted into a photon in field E_b (E_a).

Preparation of maximum coherence is critical for high-fidelity SWAP operation. For $\beta L/2=\pi/2$, one can find analytical expressions for the matrix elements of $\exp(-j\bar{\bar{M}}z)$ for an arbitrary value of atomic coherence. These are $A=\rho_{22}-\rho_{11}$, $B=-2\rho_{12}$, $C=-2\rho_{12}^{*}$, and $D=\rho_{11}-\rho_{22}$. For $|\psi\rangle_{input}=|n,m\rangle$, the fidelity of the SWAP operation can be explicitly evaluated and is

$$F \equiv |\langle m,n|\psi\rangle_{output}|^2 = |B^m C^n|^2 = 2^{2(n+m)}|\rho_{12}|^{2(n+m)}. \tag{8}$$

In Fig. 2, we plot the swap fidelity as a function of the value of the atomic coherence for different photon number eigenstates. For a maximally coherent state, $|\rho_{12}|=\frac{1}{2}$, regardless of the number of photons in each mode the fidelity of the swap operation is unity. However, as the value of the atomic coherence decreases, the fidelity is substantially reduced.

We next consider a more general input state $|\psi\rangle_{input}=|\xi,\kappa\rangle$ where $|\xi\rangle=\Sigma_k c_k|k\rangle$ and $|\kappa\rangle=\Sigma_k d_k|k\rangle$. Here, the coefficients of the Fock states, c_k and d_k, are normalized such that $\Sigma|c_k|^2=\Sigma|d_k|^2=1$. For a maximally coherent medium, and for $\beta L/2=\pi/2$, the output state is then $|\psi\rangle_{output}=|\tilde{\kappa},\tilde{\xi}\rangle$ where $|\tilde{\kappa}\rangle=\Sigma_k(-1)^k\exp(jk\phi)d_k|k\rangle$ and $|\tilde{\xi}\rangle=\Sigma_k(-1)^k\exp(-jk\phi)c_k|k\rangle$. If we choose our time origin such that $\phi=\pi$, we find that the states of the two fields are again swapped.

To produce entanglement with our scheme, one can apply E_a and E_b in a superposition of two orthogonal polarizations such that one polarization component interacts with the atomic coherence, while the other polarization component does not. Then one produces a polarization entangled state at the end of the cell which is nonfactorizable.

The experimental parameters to observe these effects is modest. For an alkili vapor cell with parameters $N=10^{12}$ cm^{-3}, $\Delta\omega=1$ GHz, and $\mu_{14}=\mu_{24}=1$ atomic unit, the necessary cell length to perform the SWAP operation is $L=7$ cm. If we assume the radiative linewidth of 1 MHz for state $|4\rangle$, the absorption of the weak fields for these parameters is less than 1% and is negligible. The absorptive effects can be further reduced by increasing $\Delta\omega$ at the expense of an increase in the density length product. Preparation of maximally coherent atoms in alkili samples require laser intensities on the order of 10 mW cm^{-2} [22]. The necessary density length products to observe some of these effects can also be achieved with cold atomic clouds in magneto optical traps [10,12]. To reduce nonadiabatic corrections to the dark state of Eq. (3), pulses long when compared to the inverse of the Rabi frequencies of the driving lasers have to be used. To avoid dephasing effects, pulses shorter than the dephasing time of the Raman transition (which can be as long as several seconds) are required.

In conclusion, we have suggested a scheme to perform a SWAP gate between photons at different wavelengths. Although we have considered an atomic system, our scheme can also be implemented using maximally coherent molecular systems [15,16]. Large Raman frequency of the molecules can enable SWAP operation between two photons at very different wavelengths, for example a red photon and a blue photon. We believe our scheme will find applications in all optical quantum computing architecture.

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