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Noise in refractive index enhancement

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By utilizing the interference between an absorptive resonance and an amplifying resonance, one can achieve an enhanced refractive index without an increase in absorption to the beam. We analyze noise added to the beam due to spontaneous emission while propagating through such an index enhanced medium. We find that, for a medium with a refractive index of n and of length L, $\approx(n-1)(2\pi/\lambda_0)L$ noise photons are added to the beam.

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Since the pioneering work of Scully and colleagues [1–6], there has been a growing interest in approaches that achieve an enhanced refractive index for a laser beam without an increase in the absorption. A key practical application of these techniques is to optical imaging science. The wavelength of light inside a refractive medium is $\lambda=\lambda_0/n$, where λ_0 is the wavelength in free space and n is the refractive index. A large refractive index, therefore, corresponds to a reduced wavelength inside the medium and enhanced imaging resolution.

It is well known that the interference of an absorptive resonance and an amplifying resonance can lead to an enhanced refractive index with vanishing absorption [3]. As shown in Fig. 1(a), the most straightforward way to realize such an interference is to have two different two-level atomic species. In practice, such a multiple two-level atom scheme has not yet been realized since it is difficult to find two different atomic species with close and easily tunable resonance frequencies. It has recently been suggested that such a multiple two-level atom scheme can be realized by using Raman resonances in far-off-resonant atomic systems [7,8]. As shown in Fig. 1(b), with an atom starting in the ground state $|1\rangle$, a Raman transition involves absorption of one photon and emission of another photon of different frequency such that the two-photon resonance condition is satisfied. By changing the order at which the probe laser, E_p, is involved in the process, such a Raman resonance can be made absorptive or amplifying. Choosing the frequencies of the two control lasers, E_g and E_l, allows precise tuning of the two resonances. Figure 1(c) shows the real part, χ', and the imaginary part, χ'', of the susceptibility as a function of the frequency of the probe laser beam. The refractive index is related to the real part through the relation $n=\sqrt{1+\chi'}$ and the imaginary part determines the loss or gain on the beam. In Fig. 1(c), for simplicity, the two Raman transitions are assumed to have equal parameters including an identical Raman linewidth of γ. In the plots, the spacing between the two Raman transitions is $\Delta=10\gamma, 5\gamma, \gamma$, respectively. At the midpoint between the two resonances, the beam experiences an enhanced refractive index with vanishing absorption. Recently, this idea has been experimentally demonstrated by using two Raman transitions in two isotopes of atomic rubidium (Rb) in a vapor cell [9].

A key question in this scheme is the noise added to the beam while propagating through such an index enhanced medium. At the midpoint between the resonances, although the beam on average experiences vanishing absorption or gain, due to spontaneously emitted photons, the beam becomes noisier as it travels through the medium. In this paper, by using the Heisenberg-Langevin approach, we estimate the number of noise photons added to the beam. We find that, at the point of vanishing absorption, roughly $(n-1)(2\pi/\lambda_0)L$ noise photons are added to the beam at the end of a medium of length L.

Before we proceed with a detailed analysis, we note that to obtain a large refractive index, it is critical that the two resonances are close to each other and interfere strongly [7]. Figure 2 shows this result. Here, we plot the real part of the

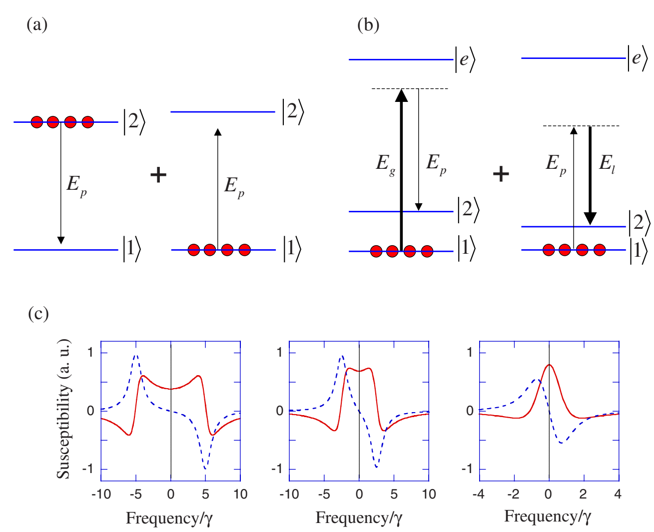

FIG. 1. (Color online) The interference of an absorptive resonance and an amplifying resonance can lead to an enhanced refractive index with vanishing absorption. (a) The most straightforward way to achieve such an interference. Due to various difficulties, the scheme in (a) is not practical. (b) An equivalent scheme that uses Raman transitions in far-off-resonant atomic systems. With an atom starting in the ground state $|1\rangle$, a Raman transition involves absorption of one photon and emission of another photon of different frequency such that the two-photon resonance condition is satisfied. By changing the order at which the probe laser, E_p, is involved in the process, such a Raman resonance can be made absorptive or amplifying. (c) The real part χ' (solid line) and the imaginary part χ'' (dashed line) of the susceptibility as a function of frequency. Between plots, the spacing between the two resonances is varied.

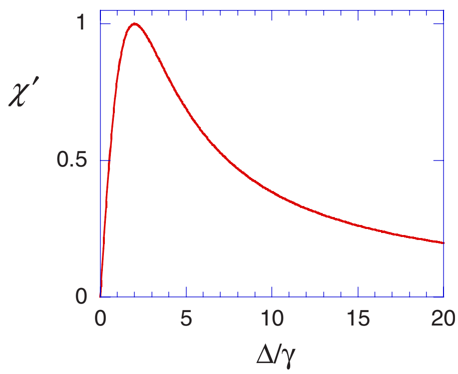

FIG. 2. (Color online) The real part of the susceptibility, χ', as a function of the separation of the two resonances, Δ, at the point of vanishing absorption. The real part of the susceptibility, χ', gets its largest value for $\Delta = 2\gamma$.

susceptibility as a function of the separation of the two resonances, Δ, at the point of vanishing absorption (the midpoint between the resonances). The real part of the susceptibility, χ', gets its largest value for $\Delta = 2\gamma$. For $\Delta \gg \gamma$, the system becomes two isolated Raman resonances and χ' drops. In the other limit of $\Delta \ll \gamma$, the effects of the two Raman resonances cancel each other resulting again in a small value of χ'. Throughout this paper, we will mostly consider the region where the two resonances interfere strongly ($\Delta \approx 2\gamma$) and therefore the maximum refractive index is achieved.

We proceed with the Heisenberg-Langevin analysis of the system of Fig. 1(b). We consider the interaction of a weak probe beam and two strong control lasers with two different atomic species [species g (gain) and species l (loss)]. Throughout this paper, we will take the two control laser beams, E_g and E_l, to be sufficiently strong such that they can be treated classically. We will also assume the atomic medium to be sufficiently dilute such that the refractive index is not too different from unity and the slowly varying envelope approximation is valid. When the laser beams are far-detuned from the excited electronic state $|e\rangle$, the Raman interaction can be modeled as an effective two level system. Within this approximation, the following Hamiltonian describes the interaction of the atoms with the fields [7,10]:

$$\hat{H}_{int} = -\hbar \sum_{i=1}^{N_g} (-\delta\omega_g \hat{\sigma}^i_{g,22} + \kappa_g \hat{a} \hat{\sigma}^i_{g,12} + \kappa_g^* \hat{a}^\dagger \hat{\sigma}^{i\dagger}_{g,12}) - \hbar \sum_{j=1}^{N_l} (-\delta\omega_l \hat{\sigma}^j_{l,22} + \kappa_l \hat{a} \hat{\sigma}^{j\dagger}_{l,12} + \kappa_l^* \hat{a}^\dagger \hat{\sigma}^j_{l,12}). \quad (1)$$

Here, $\hat{a}$ is the slowly varying annihilation operator for the probe laser beam with the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$. N_g and N_l are the number of atoms in each species and the quantities $\delta\omega_g$ and $\delta\omega_l$ are two-photon detunings from the Raman resonances, $\delta\omega_g = (\omega_{g,2} - \omega_{g,1}) - (\omega_g - \omega_p)$, $\delta\omega_l = (\omega_{l,2} - \omega_{l,1}) - (\omega_p - \omega_l)$. Throughout this paper, we will use the indices i and j to denote the individual atoms in gain and loss species, respectively. The slowly varying atomic operators for individual atoms are defined as

$$\hat{\sigma}^i_{g,11} = |1\rangle^i_g\langle 1|, \quad \hat{\sigma}^i_{g,22} = |2\rangle^i_g\langle 2|,$$

$$\hat{\sigma}^i_{g,12} = |1\rangle^i_g\langle 2| \exp[-i(\omega_{g,1} - \omega_{g,2} + \delta\omega_g)t],$$

$$\hat{\sigma}^j_{l,11} = |1\rangle^j_l\langle 1|, \quad \hat{\sigma}^j_{l,22} = |2\rangle^j_l\langle 2|, \quad (2)$$

$$\hat{\sigma}^j_{l,12} = |1\rangle^j_l\langle 2| \exp[-i(\omega_{l,1} - \omega_{l,2} + \delta\omega_l)t].$$

In Eq. (1), the constants κ_g and κ_l determine the Raman coupling and they are

$$\kappa_g = \frac{\mu_{g,1e}\mu_{g,2e}\varepsilon E_g^*}{2\hbar^2(\omega_{g,e2} - \omega_p)}, \quad \kappa_l = \frac{\mu_{l,1e}\mu_{l,2e}\varepsilon E_l^*}{2\hbar^2(\omega_{l,e1} - \omega_p)}, \quad (3)$$

where the quantities μ are the dipole matrix elements between relevant states, and $\varepsilon = \sqrt{\frac{\hbar\omega_p}{2\epsilon_0 V}}$ is the electric field due to a single probe photon (V is the quantization volume). Using Eq. (1) we can write the Heisenberg equations of motion for the atomic operators $\hat{\sigma}^{i(j)}_{g(l),\mu\nu}$ and the field operator $\hat{a}$. The equations for the off-diagonal (coherence) atomic operators and the annihilation operator for the probe beam are

$$\frac{d\hat{\sigma}^i_{g,12}}{dt} = i\kappa_g^* \hat{a}^\dagger (\hat{\sigma}^i_{g,11} - \hat{\sigma}^i_{g,22}) - (\gamma_g + i\delta\omega_g)\hat{\sigma}^i_{g,12} + \hat{f}^i_{g,12},$$

$$\frac{d\hat{\sigma}^j_{l,12}}{dt} = i\kappa_l \hat{a} (\hat{\sigma}^j_{l,11} - \hat{\sigma}^j_{l,22}) - (\gamma_l + i\delta\omega_l)\hat{\sigma}^j_{l,12} + \hat{f}^j_{l,12},$$

$$\frac{d\hat{a}}{dt} = i\kappa_g^* \sum_{i=1}^{N_g} \hat{\sigma}^{i\dagger}_{g,12} + i\kappa_l^* \sum_{j=1}^{N_l} \hat{\sigma}^j_{l,12}. \quad (4)$$

Here, γ_g and γ_l are the Raman linewidths (including dephasing and population decay) and the operators $\hat{f}$ are the Langevin fluctuation (noise) operators that accompany dissipation. We take the fluctuation operators to be Markovian with δ time correlation functions. These correlation functions can be found by using the generalized fluctuation-dissipation theorem (Einstein's relation) and are [11,12]

$$\langle \hat{f}^{i\dagger}_{g,12}(t)\hat{f}^i_{g,12}(t')\rangle = (2\gamma_g - \Gamma_g)\langle\hat{\sigma}^i_{g,22}\rangle\delta(t-t'),$$

$$\langle \hat{f}^i_{g,12}(t)\hat{f}^{i\dagger}_{g,12}(t')\rangle = [2\gamma_g\langle\hat{\sigma}^i_{g,11}\rangle + \Gamma_g\langle\hat{\sigma}^i_{g,22}\rangle]\delta(t-t'),$$

$$\langle \hat{f}^{j\dagger}_{l,12}(t)\hat{f}^j_{l,12}(t')\rangle = (2\gamma_l - \Gamma_l)\langle\hat{\sigma}^j_{l,22}\rangle\delta(t-t'),$$

$$\langle \hat{f}^j_{l,12}(t)\hat{f}^{j\dagger}_{l,12}(t')\rangle = [2\gamma_l\langle\hat{\sigma}^j_{l,11}\rangle + \Gamma_l\langle\hat{\sigma}^j_{l,22}\rangle]\delta(t-t'). \quad (5)$$

In the above, Γ_g and Γ_l are the decay rates of the populations from state $|2\rangle$ to state $|1\rangle$ in each species, respectively.

We proceed with an analysis of the Heisenberg-Langevin equations [Eqs. (4)] in the perturbative and adiabatic limit. We take the probe beam to be sufficiently weak such that the atomic populations stay mostly in the ground state (state $|1\rangle$) and take $\hat{\sigma}^i_{g,11} = \hat{\sigma}^j_{l,11} \approx 1$, $\hat{\sigma}^i_{g,22} = \hat{\sigma}^j_{l,22} \approx 0$. We also make the adiabatic approximation and assume the rate of change of the annihilation operator to be much smaller when compared with the Raman linewidths, $d\hat{a}/dt \ll \gamma_g \hat{a}, \gamma_l \hat{a}$ [13–15]. Adiabatic approximation requires the atomic medium to be sufficiently dilute such that the gain and loss rates on the probe beam are much smaller when compared with the Raman linewidths. With these assumptions, the solutions of Eq. (4) for the atomic coherence operators are

$$\hat{\sigma}^j_{g,12}(t) = i\frac{\kappa_g^*}{\gamma_g + i\delta\omega_g}\hat{a}^\dagger + \int_0^t \exp[-(\gamma_g + i\delta\omega_g)(t-t')]\hat{f}^i_{g,12}(t')dt',$$

$$\hat{\sigma}^j_{l,12}(t) = i\frac{\kappa_l}{\gamma_l + i\delta\omega_l}\hat{a} + \int_0^t \exp[-(\gamma_l + i\delta\omega_l)(t-t')]\hat{f}^j_{l,12}(t')dt'. \quad (6)$$

In Eq. (6), we do not make any approximation with regard to the Langevin fluctuation operators since these operators have very fast time variation. We then use the solution of Eqs. (6) in the Heisenberg equation of motion for the probe annihilation operator and reduce the problem to a single differential equation:

$$\frac{d\hat{a}}{dt} = (g-l)\hat{a} + i(\beta_g + \beta_l)\hat{a} + \hat{F}_g + \hat{F}_l. \quad (7)$$

Here, g is the gain coefficient experienced by the probe beam due to Raman excitation in the first species and l is the loss coefficient due to Raman excitation in the second species. The quantities β_g and β_l are the phase accumulation rates (which will determine the refractive index) due to each Raman interaction, respectively. The expressions for these quantities are

$$g = \frac{|\kappa_g|^2 N_g}{\gamma_g^2 + \delta\omega_g^2}\gamma_g, \quad \beta_g = \frac{|\kappa_g|^2 N_g}{\gamma_g^2 + \delta\omega_g^2}\delta\omega_g,$$

$$l = \frac{|\kappa_l|^2 N_l}{\gamma_l^2 + \delta\omega_l^2}\gamma_l, \quad \beta_l = \frac{|\kappa_l|^2 N_l}{\gamma_l^2 + \delta\omega_l^2}\delta\omega_l. \quad (8)$$

In Eq. (7), the quantities $\hat{F}_g$ and $\hat{F}_l$ are collective noise operators that accompany gain and loss and they are

$$\hat{F}_g(t) = i\kappa_g^* \sum_{i=1}^{N_g} \int_0^t \exp[-(\gamma_g - i\delta\omega_g)(t-t')]\hat{f}^{i\dagger}_{g,12}(t')dt',$$

$$\hat{F}_l(t) = i\kappa_l^* \sum_{j=1}^{N_l} \int_0^t \exp[-(\gamma_l + i\delta\omega_l)(t-t')]\hat{f}^j_{l,12}(t')dt'. \quad (9)$$

Using Eqs. (9) and (5), we derive the time correlations of the collective noise operators:

$$\langle \hat{F}_g^\dagger(t)\hat{F}_g(t')\rangle = |\kappa_g|^2 N_g \exp(-\gamma_g|t-t'|)\exp[-i\delta\omega_g(t-t')],$$

$$\langle \hat{F}_l(t)\hat{F}_l^\dagger(t')\rangle = |\kappa_l|^2 N_l \exp(-\gamma_l|t-t'|)\exp[-i\delta\omega_l(t-t')]. \quad (10)$$

The collective noise operators have exponentially decaying correlations as a function of the time difference. All other time correlations between the collective noise operators vanish. Using Eqs. (10) and the formal solution of Eq. (7), we can derive the time correlations between the probe annihilation operator and the collective noise operators. Within the adiabatic approximation, these correlations are

$$\langle \hat{a}^\dagger(t)\hat{F}_g(t)\rangle = \langle \hat{F}_g^\dagger(t)\hat{a}(t)\rangle^\dagger = \frac{|\kappa_g|^2 N_g}{\gamma_g - i\delta\omega_g},$$

$$\langle \hat{a}(t)\hat{F}_l^\dagger(t)\rangle = \langle \hat{F}_l(t)\hat{a}^\dagger(t)\rangle^\dagger = \frac{|\kappa_l|^2 N_l}{\gamma_l - i\delta\omega_l}. \quad (11)$$

As expected, using Eqs. (7) and (11), it can be shown that the commutator relation for the probe annihilation operator is preserved at all times, $[\hat{a}(t), \hat{a}^\dagger(t)] = 1$.

We next evaluate the noise added to the beam due to spontaneous emission while propagating through an index-enhanced medium. For this purpose, we define the number of photons in the probe beam, $\zeta(t) = \langle \hat{a}^\dagger(t)\hat{a}(t)\rangle$. Using the probe field differential equation [Eq. (7)] and the time correlations between the probe annihilation operator and the noise operators [Eqs. (11)], the evolution equation for the number of photons is

$$\frac{d\zeta}{dt} = 2(g-l)\zeta + 2g. \quad (12)$$

Equation (12) can be solved analytically and its solution is

$$\zeta(t) = \exp[2(g-l)t]\zeta(0) + \frac{g}{g-l}\{\exp[2(g-l)t] - 1\} \equiv \exp[2(g-l)t]\zeta(0) + \zeta_{\text{noise}}(t). \quad (13)$$

The first term in Eq. (13) is the change in the number of photons due to coherent amplification (or absorption) of the beam. The second term is the number of added noise photons to the beam due to spontaneous emission.

We proceed with a discussion of the refractive index and its relation to the amount of noise. For this purpose, we consider a free-space traveling wave geometry with beams propagating in the z direction and replace the time differential equations with spatial equations, $\frac{d}{dt} \to c\frac{d}{dz}$, in Eqs. (7) and (12) [16,17]. The classical limit of Eq. (7) can be obtained by replacing the annihilation operator with its average value $a = \langle \hat{a}\rangle$. In this limit, the fluctuations can be ignored and we obtain $a(z) = a(0)\exp\{[(g-l)/c]z + i[(\beta_g+\beta_l)/c]z\}$. The refractive index for the beam is $n = 1 + [(\beta_g+\beta_l)/c](\lambda_0/2\pi)$. The susceptibility curves of Fig. 1(c) are valid in this limit.

When the fluctuations are included, the increased refractive index comes at the cost of added noise to the beam. From Eq. (13), the number of noise photons is $\zeta_{\text{noise}}(z) = \frac{g}{g-l}(\exp\{[2(g-l)/c]z\} - 1)$. We next evaluate the number of noise photons for the ideal case of equal parameters for the two Raman transitions: $\gamma_g = \gamma_l$, $N_g = N_l$, $E_g = E_l$, and $\mu_g = \mu_l$.

For this case, as shown in Fig. 1(c), the vanishing absorption point occurs at the midpoint between the resonances and the maximum refractive index is obtained for $\Delta = 2\gamma_g$. For these conditions, we have

$$g = l = \frac{|\kappa_g|^2 N_g}{2\gamma_g} = \beta_g = \beta_l. \tag{14}$$

For the conditions of Eq. (14), the number of noise photons added to the beam is

$$\zeta_{\text{noise}}(z) = 2(g/c)z = [(\beta_g + \beta_l)/c]z = (n-1)\frac{2\pi}{\lambda_0}z. \tag{15}$$

Equation (15) is the central result of this work. At the point of vanishing absorption, the number of noise photons that is added to the probe beam equals the phase accumulation due to enhanced refractive index. As an example, Eq. (15) suggests that for $n = 1.1$ and $L = 1$ mm, the number of noise photons at the end of the index-enhanced medium will be about 10^3 and will be negligible for a microwatt level probe laser beam.

The result of Eq. (15) is valid for the case when the two resonances interfere strongly, $\Delta \approx 2\gamma_g$, and therefore the maximum refractive index is achieved. As the spacing between the two resonances becomes large, $\Delta \gg \gamma_g$, we have the usual scaling for the refractive index and the dissipative processes that determine noise. For $\Delta \gg \gamma_g$, the achieved refractive index at the point of vanishing absorption drops as $\sim 1/\Delta$, whereas the noise drops as $\sim \gamma_g/\Delta^2$. As a result, Eq. (15) reads $\zeta_{\text{noise}}(z) \approx (n-1)(\gamma_g/\Delta)(2\pi/\lambda_0)z$.

We note that Eq. (15) is the number of noise photons whose frequency overlaps with the probe laser beam which is tuned to the midpoint of the two resonances. However, the amplification of spontaneously emitted photons whose frequency lies in the gain region is a concern (amplified spontaneous emission). This effect is common to almost all index-enhancement schemes and may practically limit the achievable length of the index-enhanced medium. The noise photons due to amplified spontaneous emission will have a different frequency when compared with the probe laser beam. As a result, after the probe laser leaves the index-enhanced medium, these photons can be filtered out by using a narrow-band transmission filter (for example, with a high finesse cavity that is locked to the probe laser frequency). Furthermore, this drawback can, in principle, be overcome by suppressing the vacuum modes that lie in the gain region of the susceptibility curves. If the vacuum modes are suppressed, the spontaneous emission at those frequencies will be prohibited. This can, for example, be achieved by placing the index-enhanced medium inside a cavity.

We stress, once again, that throughout this paper we have assumed the atomic medium to be dilute. When this assumption is not satisfied, the slowly varying envelope approximation breaks down and our formalism is no longer valid. If the collisional processes can be ignored, we expect Eq. (15) to apply even when the dilute medium assumption is not satisfied. This is because the noise is due to spontaneous emission and we do not expect any new physics to come into play as long as collisions do not alter the response of each atom significantly.

In conclusion we have analyzed the noise added to a beam while propagating through an index-enhanced medium that utilizes the interference of an absorptive and an amplifying resonance. We note that the results of this paper can also be derived by using appropriately averaged position dependent operators with spatial derivatives introduced at the start of the formalism [13–15,18]. Although we have not proven yet, we suspect that a result similar to Eq. (15) will apply to other index-enhancement schemes [1–4]. Since all of these schemes rely on atomic excitation to the excited state, they must suffer from spontaneous emission to some extent. Proving a more general result in the spirit of Eq. (15) will be among our future investigations.

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